

# A Kalman-filter-based approach to combining independent Earth-orientation series

R. S. Gross, T. M. Eubanks\*, J. A. Steppe, A. P. Freedman, J. O. Dickey, T. F. Runge

Jet Propulsion Laboratory, California Institute of Technology, Mail stop 238-332, 4800 Oak Grove Drive, Pasadena, CA 91109, USA  
Tel: +1 818 354 4010; fax: +1 818 393 6890; e-mail: rsg@logos.jpl.nasa.gov

Received: 28 January 1997 / Accepted: 20 August 1997

**Abstract.** An approach, based upon the use of a Kalman filter, that is currently employed at the Jet Propulsion Laboratory (JPL) for combining independent measurements of the Earth's orientation, is presented. Since changes in the Earth's orientation can be described as a randomly excited stochastic process, the uncertainty in our knowledge of the Earth's orientation grows rapidly in the absence of measurements. The Kalman-filter methodology allows for an objective accounting of this uncertainty growth, thereby facilitating the intercomparison of measurements taken at different epochs (not necessarily uniformly spaced in time) and with different precision. As an example of this approach to combining Earth-orientation series, a description is given of a combination, SPACE95, that has been generated recently at JPL.

**Key words.** Earth rotation · Combination · Kalman filter

## 1 Introduction

A new field in the geophysical sciences has recently emerged, namely, that of space geodesy. An integral part of geodesy has always been the definition and realization of a terrestrial, body-fixed reference frame, a celestial, space-fixed reference frame, and the determination of the Earth-orientation parameters (precession, nutation, spin, and polar motion) that link these two reference frames together. But with the advent of space geodesy – with the placement of laser retro-reflectors on the Moon by Apollo astronauts and Soviet landers, the launch of the Laser Geodynamics Satellite (LAGEOS), and the development of very long baseline interferometry and

the global positioning system – a quantum leap has been taken in our ability to realize the terrestrial and celestial reference frames and to determine the Earth-orientation parameters.

Each of the modern space-geodetic techniques of lunar laser ranging (LLR), satellite laser ranging (SLR), very long baseline interferometry (VLBI), and the global positioning system (GPS) is able to determine the Earth-orientation parameters. But each technique has its own unique strengths and weaknesses in this regard. Not only is each technique sensitive to a different subset and/or linear combination of the Earth-orientation parameters, but the averaging time for their determination is different, as is the interval between observations and the precision with which they can be determined. By combining the individual Earth-orientation series determined by each technique, a series of the Earth's orientation can be obtained that is based upon independent measurements and that spans the greatest possible time-interval. Such a combined Earth-orientation series is useful for a number of purposes, including a variety of scientific studies, and as an a priori series for use in data reduction procedures. However, care must be taken in generating such a combined series in order to account for differences in the underlying reference frames within which each individual series is determined (which can lead to differences in bias and rate between the Earth-orientation series), as well as to properly assign relative weights to each observation prior to combination. The issues and concerns surrounding the combination of Earth-orientation series is the subject of this report (see also Gross 1996a). As a framework in which to discuss these issues and concerns, a description will be given of the determination of a particular combination, SPACE95 (Gross 1996b), that has been generated recently at the Jet Propulsion Laboratory (JPL) from the space-geodetic Earth-orientation series listed in Table 1. Other approaches to combining Earth-orientation series have been discussed by Vicente and Wilson (1986), Archinal (1988), Babcock (1988), Wilson and Vicente (1988), McCarthy and Luzum (1991, 1996), and IERS (1995, Sect. II-4.1).

Correspondence to: Richard S. Gross

\*T.M. Eubanks now at US Naval Observatory, Washington, DC

**Table 1.** Data sets combined to form SPACE95

data set name	data type	analysis center	data span	number used
LLR (JPL95M01; VOL, UT0)				
McDonald Cluster	LLR	JPL	05Oct76–28Dec95	485
CERGA	LLR	JPL	07Apr84–14Dec95	538
Haleakala	LLR	JPL	10Feb85–11Aug90	65
DSN (JPL95R01; T, V)				
California–Spain Cluster	VLBI	JPL	26Nov79–06Jan96	578
California–Australia Cluster	VLBI	JPL	28Oct78–09Jan96	581
SGP/CDP (GLB973f; T, V)				
Westford–Ft. Davis	VLBI	GSFC	25Jun81–01Jan84	103
Westford–Mojave	VLBI	GSFC	21Mar85–06Aug90	13
SGP/CDP (GLB973f; UTPM)				
Multibaseline	VLBI	GSFC	04Aug79–28Dec94	1784
USNO (N9604 15Feb96; UTPM)				
Multibaseline	VLBI	USNO	29Dec94–07Feb96	106
NOAA (NOAA95R02; UT1)				
IRIS Intensive	VLBI	NOAA	02Apr84–31Dec94	2356
USNO (N9604.INT 14Feb96; UT1)				
IRIS Intensive	VLBI	USNO	04Jan95–10Feb96	269
UTCSR (CSR95L01; PMX, PMY)				
Lageos	SLR	UTCSR	02Oct76–28Jan95	2025
GPS (SIO93P01; PMX, PMY)				
Scripps	GPS	SIO	25Aug91–31May92	265
GPS (JPL95P02; PMX, PMY)				
JPL	GPS	JPL	01Jun92–27Jan95	817
GPS (IGS Analysis; PMX, PMY)				
JPL	GPS	JPL	28Jan95–10Feb96	365

## 2 Space-geodetic Earth orientation series

### 2.1 Satellite laser ranging

In the technique of SLR, the round-trip times of flight of laser light pulses are accurately measured as they are emitted from a laser system located at some ground-based observing station, travel through the Earth's atmosphere to some artificial satellite orbiting the Earth, are reflected by retro-reflectors carried onboard that satellite, and return to the same observing station from which they were emitted (e.g., Lambeck 1988, Chap. 6). This time-of-flight range measurement is converted into a distance measurement by using the speed of light and correcting for a variety of known or modeled effects, such as atmospheric path delay and satellite center-of-mass offset. Although a number of satellites carry retro-reflectors for tracking and navigation purposes, the LAGEOS satellite is most commonly used for the determination of the Earth-orientation parameters, since it was specifically designed and launched to study this and other geodetic properties of the Earth (e.g., Christodoulidis et al. 1985; Cohen and Smith 1985).

The Earth-orientation parameters are recovered from the basic range measurements in the course of determining the satellite's orbit. The basic range measurement is sensitive to any geophysical process which changes the distance between the satellite and the observing station, such as displacements of the satellite due

to perturbations of the Earth's gravitational field, motions of the observing station due to tidal displacements or plate tectonics, or a change in the orientation of the Earth (which changes the location of the observing station with respect to the satellite). These and other geophysical processes must be modeled when fitting the satellite's orbit to the range measurements as obtained at a number of globally distributed tracking stations. Adjustments to the a priori models used for these effects can then be obtained during the orbit determination procedure, thereby enabling, for example, the determination of the Earth-orientation parameters (e.g., Smith et al. 1985, 1990, 1991, 1994; Tapley et al. 1985, 1993).

A number of organizations, including groups at Goddard Space Flight Center (GSFC), the University of Texas at Austin Center for Space Research (UTCSR), and the Delft University of Technology (DUT), currently determine the Earth-orientation parameters from LAGEOS range measurements. Since each of these groups uses the same basic set of LAGEOS range measurements, their results are not completely independent of each other even though their subsequent data editing and processing procedures are different. For the purpose of combining Earth-orientation series, it is desirable to combine only completely independent determinations of the Earth's orientation. Thus, in generating SPACE95, only one SLR data set was used, namely, that determined at the University of Texas at Austin Center for Space Research.

The particular UTCSR SLR Earth-orientation data set used in generating SPACE95 is the series designated EOP(CSR) 95 L 01 (Eanes and Watkins 1995). This series consists of values for universal time (UT1) and the  $x$  and  $y$  components of polar motion (PMX and PMY, respectively) spanning 19 May 1976 to 28 January 1995. However, because the first few values of this series have larger-than-usual stated uncertainties and averaging intervals, only those values after 2 October 1976 were used in generating SPACE95. Also, only the polar motion values of this series have been used in generating SPACE95. The UT1 determinations were not incorporated into SPACE95, since their long-period behavior has been constrained to that of the *a priori* series (Eanes and Watkins 1995), and hence they are not entirely based upon range measurements as the PMX and PMY values are. It is difficult to separate variations in UT1 from variations in the orbital node of the LAGEOS satellite due to the effect of unmodeled forces acting on the satellite (e.g., Lambeck 1988, Sect. 6.3.5), so that both these quantities cannot be simultaneously estimated without making additional assumptions. Solutions for these quantities are obtained by assuming that the effects of the unmodeled forces are such that they cause the orbital node to vary slowly, and that rapid variations therefore reflect UT1 behavior. Hence, the slow UT1 variations are not adjusted, but are constrained to those of the *a priori* series. The Kalman filter used at JPL to combine Earth-orientation series (discussed in the following) does not currently have the capability of separating the measured rapid UT1 variations from the constrained slow variations. Thus, to insure that only independent determinations are incorporated into the combined Earth-orientation series, the UTCSR SLR UT1 values have not been used. The proper incorporation into the Kalman filter of the rapid SLR UT1 variations (but not the slow variations) is currently under study.

## 2.2 Lunar laser ranging

The technique of LLR is similar to that of SLR, except that the laser retro-reflector is located on the Moon, rather than on some artificial satellite (e.g., Mulholland 1980; Lambeck 1988, Chap. 7). LLR is technically more challenging than SLR because of the need to detect the much weaker signal returned from the Moon than from the much closer artificial satellite. Larger, more powerful laser systems with more sophisticated signal detectors need to be employed in LLR; consequently, there are far fewer stations that range to the Moon than to LAGEOS. In fact, there are currently only two stations that regularly range to the Moon: the McDonald Observatory in Texas and the CERGA station in Grasse, France.

The Earth-orientation parameters are typically determined from LLR by analyzing the residuals at each station after the lunar orbit (and other parameters such as station and reflector locations) has been fit to the range measurements from all the stations (e.g., Stolz

et al. 1976; Langley et al. 1981; Dickey et al. 1985, 1994; Newhall et al. 1988; Williams et al. 1993; Whipple 1993). From this single-station technique, two linear combinations of UT1 and the polar motion parameters PMX and PMY can be determined, namely, UT0 and the variation of latitude (VOL) at that station. A number of organizations, including groups at the University of Texas McDonald Observatory and at JPL, currently determine UT0 and the variation of latitude from LLR. But all such determinations are based upon the same set of range measurements and are therefore not completely independent of each other, even though the subsequent data editing and processing procedures employed by each analysis center are different. Since it is desirable to combine only independent Earth-orientation results, only one LLR solution has been included in SPACE95, namely that determined at JPL.

The particular JPL LLR series used in generating SPACE95 is an updated version of EOP(JPL) 95 M 01 (Newhall et al. 1995). This series consists of values for UT0 and the variation of latitude as determined from observations taken by the LLR station at Haleakala Observatory in Hawaii (spanning 10 February 1985 to 11 August 1990), by the CERGA system in Grasse, France (spanning 7 April 1984 to 14 December 1995), and by the three LLR stations that have been located at McDonald Observatory in Texas (collectively spanning 15 April 1970 to 28 December 1995). However, in generating SPACE95, the LLR UT0 and VOL determinations made from observations taken prior to 2 October 1976 have not been used. Polar motion values are needed to convert the LLR UT0 measurements to UT1 [See Eq. (1) below]. Since regular space-geodetic measurements are not available for all three Earth-orientation components before the SLR series begins, and since only those SLR measurements since 2 October 1976 have been used in generating SPACE95 (see Table 1 and Sect. 2.1), no LLR measurements made prior to this date have been used in SPACE95.

The two Earth-orientation parameters UT0 and VOL are related to UT1 and the polar motion parameters PMX and PMY by the well-known expressions (e.g., Moritz and Mueller 1988, p. 425):

$$\Delta\phi_i(t) = x_p(t) \cos \lambda_i - y_p(t) \sin \lambda_i \quad (1a)$$

$$UT0_i(t) - TAI(t) = U(t) + x_p(t) \sin \lambda_i \tan \phi_i + y_p(t) \cos \lambda_i \tan \phi_i \quad (1b)$$

where  $\Delta\phi_i$  is the variation of latitude observed at the station  $i$  located at nominal latitude  $\phi_i$  and east longitude  $\lambda_i$ , the observed UT0 at that station is designated  $UT0_i(t) - TAI(t)$  where  $TAI(t)$  is a reference time-scale based upon atomic clocks, the variable  $U(t)$  is defined by  $U(t) \equiv UT1(t) - TAI(t)$ , and  $x_p$  and  $y_p$  are the polar motion parameters PMX and PMY, respectively, with  $y_p$  being positive towards 90°W longitude. A third linear combination  $D_i(t)$  of the UTPM parameters (PMX, PMY, UT1), representing that component of the Earth's orientation which cannot be determined from LLR observations at the single station  $i$ , is given by:

$$D_i(t) = U(t) \sin \phi_i - x_p(t) \sin \lambda_i \cos \phi_i - y_p(t) \cos \lambda_i \cos \phi_i \quad (1c)$$

This third, degenerate, component of the Earth's orientation represents changes in the orientation of the Earth resulting from a rotation of the Earth about an axis defined by the location of the station and the origin of the terrestrial reference frame (that is, about the position vector of the station).

In the Kalman filter it is necessary to be able to transform readily between the observed  $UT0$ ,  $VOL$  values and the corresponding  $UTPM$  values. This is most conveniently accomplished by means of an orthogonal transformation matrix. As written, the system Eq. (1) does not represent an orthogonal transformation, although one can be obtained upon defining a new parameter  $UTF_i$  related to the observed  $UT0_i(t) - TAI(t)$  by:

$$UTF_i(t) = \cos \phi_i [UT0_i(t) - TAI(t)] \quad (2)$$

The transformation between the  $UTPM$  parameters and the  $VUD$  parameters ( $\Delta\phi_i, UTF_i, D_i$ ) can then be written in matrix form as:

$$\begin{pmatrix} \Delta\phi_i(t) \\ UTF_i(t) \\ D_i(t) \end{pmatrix} = \begin{pmatrix} \cos \lambda_i & -\sin \lambda_i & 0 \\ \sin \lambda_i \sin \phi_i & \cos \lambda_i \sin \phi_i & \cos \phi_i \\ -\sin \lambda_i \cos \phi_i & -\cos \lambda_i \cos \phi_i & \sin \phi_i \end{pmatrix} \times \begin{pmatrix} x_p(t) \\ y_p(t) \\ U(t) \end{pmatrix} \quad (3)$$

where it is easily shown that the resulting  $3 \times 3$  transformation matrix is orthogonal. Unit vectors in the  $\Delta\phi_i$ ,  $UTF_i$ , and  $D_i$  directions of Earth-orientation parameter space form a triad of orthonormal base vectors that span this parameter space. Note that in the system of equations given by Eqs. (1) or (3), the Earth-orientation parameters are all assumed to have the same units. If the measured values [e.g.,  $UT0_i(t) - TAI(t)$  and  $\Delta\phi_i(t)$ ] have different units, then they must be converted to the same units prior to application of the transformation represented by Eq. (3).

In the Kalman filter, besides transforming between the  $UTPM$  and  $VUD$  parameters themselves, it will also be necessary to transform their respective covariance matrices. This can be accomplished using the  $3 \times 3$  transformation matrix defined by Eq. (3) once the stated uncertainties and correlations of the measurements have been converted from those appropriate for the measured  $\Delta\phi_i$  and  $UT0_i - TAI$  to those appropriate for  $\Delta\phi_i$  and  $UTF_i$ . To accomplish this (once the units of the measured  $UT0_i - TAI$  values and uncertainties have been changed to those of  $\Delta\phi_i$ ) it is only necessary to convert the stated uncertainties of the measured  $UT0_i - TAI$  to those appropriate for  $UTF_i$  by multiplying them by the same scale factor used in Eq. (2) to convert  $UT0_i - TAI$  to  $UTF_i$ , that is, by  $\cos \phi_i$ . The correlation between  $\Delta\phi_i$  and  $UTF_i$  is the same as that between  $\Delta\phi_i$  and  $UT0_i - TAI$  (correlations are unaffected by scale factor differences),

and thus no conversion of the stated correlations need be made.

### 2.3 Very long baseline interferometry

Radio interferometry is routinely used to make highly accurate measurements of changes in  $UT1$  and polar motion with observing sessions lasting from about an hour to a day. The VLBI technique measures the difference in the arrival time of a radio signal at two or more radio telescopes that are simultaneously observing the same distant source (e.g., Shapiro 1983; Lambeck 1988, Chap. 8). This technique is therefore sensitive to processes that change the relative position of the radio telescopes with respect to the source, such as a change in the orientation of the Earth in space or a change in the position of the telescopes due to, for example, tidal displacements or tectonic motions. If just two telescopes are observing the same sources, then only two components of the Earth's orientation can be determined. A rotation of the Earth about an axis parallel to the baseline connecting the two radio telescopes does not change the relative position of the telescopes with respect to the sources, and hence this component of the Earth's orientation is not determinable from VLBI observations taken on that single baseline. Multibaseline VLBI observations with satisfactory geometry can determine all three components of the Earth's orientation.

A number of organizations, including groups at Goddard Space Flight Center, the Jet Propulsion Laboratory, the US Naval Observatory, and the Geodetic Institute of the University of Bonn, currently determine Earth-orientation parameters from VLBI measurements using their own independent data processing and reduction procedures. However, since many of the resulting Earth-orientation series are based upon the same set of VLBI measurements, they are not entirely independent of each other. Since it is desirable to combine only independently determined Earth-orientation parameters, only those series described in the following have been incorporated into SPACE95.

**SGP/CDP.** The VLBI group of the National Aeronautics and Space Administration (NASA) Space Geodesy Program [SGP; formerly the Crustal Dynamics Project (CDP)] at Goddard Space Flight Center conducts VLBI observing campaigns in order to study the Earth's deformation and rotation (Clark et al. 1985, 1987; Ryan et al. 1986, 1993; Ma et al. 1993, 1994, 1995). Using independent processing techniques, they also reduce data taken under other observing programs, including the National Earth Orientation Service (NEOS) and the IRIS/POLARIS programs described in the coming paragraphs. The particular SGP/CDP Earth-orientation series used in generating SPACE95 is designated by them as GLB973f (Ma, personal communication, 1995); it spans 4 August 1979 to 28 December 1994 and consists of Earth-orientation parameters determined from both single-baseline and multibaseline VLBI observations.

*USNO.* As part of its participation in the National Earth Orientation Service (Eubanks et al. 1994), the US Naval Observatory (USNO) conducts VLBI observing sessions in order to regularly monitor changes in the Earth's orientation. Series of Earth-orientation parameters determined from these observations, as well as from observations conducted under other observing programs such as the SGP/CDP and IRIS/POLARIS (see next paragraph) programs, are produced by the US Naval Observatory using their own data editing and reduction procedures (Eubanks et al. 1994). The particular USNO Earth-orientation series incorporated into SPACE95 is their "n9604.eop" series dated 15 February 1996. However, since this USNO Earth-orientation series is not entirely independent of the SGP/CDP GLB973f series because they are both largely based upon the same set of VLBI measurements, only that portion of the USNO series that does not overlap with the SGP/CDP GLB973f series was incorporated into SPACE95, namely only that portion spanning 29 December 1994 to 7 February 1996.

*IRIS/POLARIS.* The Polar motion Analysis by Radio Interferometric Surveying project (POLARIS) was originally organized to monitor Earth rotation and orientation using VLBI observations within the United States (Carter 1979; Carter and Strange 1979; Carter et al. 1979, 1984; Robertson and Carter 1982). The POLARIS network was later expanded to include further European involvement in the International Radio Interferometric Surveying (IRIS) project (Carter and Robertson 1984, 1986a,b; Robertson and Carter 1985; Carter et al. 1985, 1988; Robertson et al. 1988). In addition to these 24-h sessions, regular single-baseline observing sessions of 1-h duration, the "Intensive" sessions originally using the Westford (Massachusetts) and Wettzell (Germany) radio telescopes but currently using those at Green Bank (West Virginia) and Wettzell (Robertson et al. 1985), are conducted at quasi-daily intervals for the purpose of determining UT1 (the "Intensive" sessions are not currently conducted on Sundays, nor on the day of the 24-h multibaseline NEOS session). Since the SGP/CDP GLB973f and USNO "n9604.eop" Earth-orientation series include values determined from the IRIS/POLARIS 24-h VLBI measurements, they are automatically included in SPACE95 and no separate IRIS/POLARIS series needs to be incorporated. However, since neither the SGP/CDP GLB973f nor the USNO "n9604.eop" Earth-orientation series include the "Intensive" UT1 values, they have been included in SPACE95 by incorporating series of them determined at both the USNO and the National Oceanic and Atmospheric Administration (NOAA). The particular NOAA "Intensive" UT1 series chosen for incorporation into SPACE95, EOP(NOAA) 95 R 02 (Ray et al. 1995), spans 2 April 1984 through 31 December 1994. In addition, that portion of the USNO "Intensive" UT1 series "n9604.eop.int" dated 14 February 1996 that does not overlap with the NOAA "Intensive" UT1 series, namely that portion spanning 4 January 1995 through 10 February 1996, was incorporated into SPACE95. Note that in principle the IRIS

"Intensive" measurements, being derived from single-baseline VLBI observations, should be treated as measurements of the transverse ( $T$ ) and vertical ( $V$ ) components of the Earth's orientation (see below) rather than as measurements of UT1. However, as released by both NOAA and the USNO, the IRIS "Intensive" measurements are given as measurements of UT1 without the necessary ancillary information (see below) that would allow them to be more properly treated as measurements of  $T$  and  $V$ . Hence, in SPACE95 they have been treated as UT1 measurements.

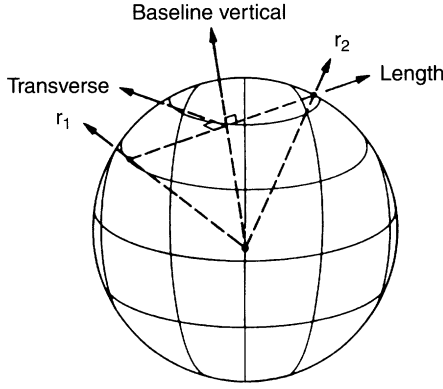
*DSN.* The Deep Space Network (DSN) of NASA has conducted single-baseline VLBI observations for the purpose of radio-source Catalog Maintenance and Enhancement (here called the CAT M&E sessions) at irregular intervals since late 1978 (Fanselow et al. 1979; Sovers et al. 1984; Steppe et al. 1995). These sessions are nominally 24 h in duration and are scheduled within a few days of each other on both the 8400-km-long baseline between the Goldstone (California) and Madrid (Spain) telescopes, and the 10 700-km-long baseline between the Goldstone and Canberra (Australia) telescopes. Since July 1980, the DSN has also undertaken single-baseline VLBI observations in support of spacecraft navigation (Eubanks et al. 1982; Steppe et al. 1995). These semiweekly Time and Earth Motion Precision Observations (TEMPO) sessions generally last no more than 3 h and are on the same baselines as are the CAT M&E sessions. The particular DSN Earth-orientation series chosen for inclusion in SPACE95, containing results determined from both the CAT M&E and TEMPO observing sessions, is a continuation of the series designated EOP(JPL) 95 R 01 (Steppe et al. 1995) and spans 28 October 1978 to 9 January 1996.

As mentioned already, only two components of the Earth's orientation can be determined from VLBI observations taken on a particular single baseline. The Kalman filter must be able to transform between these two determinable components, each of which is a particular linear combination of the usual UTPM parameters, and the UTPM parameters themselves. In order to derive the transformation matrix needed to accomplish this, first consider a special coordinate system, the transverse, vertical, length system, that has been found useful in analyzing single baseline VLBI results (Eubanks and Steppe 1988). Figure 1 illustrates this coordinate system, the basis vectors of which lie along the three mutually orthogonal directions given by the baseline length, the baseline vertical, and the baseline transverse directions. Let  $\mathbf{r}_1$  and  $\mathbf{r}_2$  denote the position vectors within some specified conventional terrestrial reference frame (with origin near the center of the Earth) of the two radio telescopes located at either end of the baseline:

$$\mathbf{r}_1 = x_1\hat{\mathbf{i}} + y_1\hat{\mathbf{j}} + z_1\hat{\mathbf{k}} \quad (4a)$$

$$\mathbf{r}_2 = x_2\hat{\mathbf{i}} + y_2\hat{\mathbf{j}} + z_2\hat{\mathbf{k}} \quad (4b)$$

where that hat ( $\hat{\cdot}$ ) denotes a vector of unit length. Mutually orthogonal unit vectors  $\hat{\mathbf{t}}, \hat{\mathbf{v}}, \hat{\mathbf{b}}$  in the baseline



**Fig. 1.** Illustration of the definition of the baseline transverse  $\hat{\tau}$ , vertical  $\hat{v}$ , length  $\hat{b}$  coordinate reference frame. The positions within some rotating, body-fixed conventional terrestrial reference frame of the VLBI observing telescopes (that are located at either end of the baseline) are given by the position vectors  $\mathbf{r}_1$  and  $\mathbf{r}_2$

transverse, vertical, and length directions, respectively, can be defined from these position vectors by:

$$\hat{\mathbf{b}} = \frac{\mathbf{r}_2 - \mathbf{r}_1}{\|\mathbf{r}_2 - \mathbf{r}_1\|} \quad (5a)$$

$$\hat{\tau} = \frac{\mathbf{r}_1 \times \mathbf{r}_2}{\|\mathbf{r}_1 \times \mathbf{r}_2\|} \quad (5b)$$

$$\hat{\mathbf{v}} = \hat{\mathbf{b}} \times \hat{\tau} \quad (5c)$$

The coordinate transformation matrix relating the components  $w_\tau, w_v, w_b$  of some position vector  $\mathbf{w}$  in the baseline transverse, vertical, length system to its components  $w_x, w_y, w_z$  in the usual  $\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}$  system is then given by:

$$\begin{pmatrix} w_\tau \\ w_v \\ w_b \end{pmatrix} = \begin{pmatrix} \tau_x & \tau_y & \tau_z \\ v_x & v_y & v_z \\ b_x & b_y & b_z \end{pmatrix} \begin{pmatrix} w_x \\ w_y \\ w_z \end{pmatrix} \quad (6)$$

where expressions for the  $\tau_i, v_i$ , and  $b_i$  elements of the coordinate transformation matrix in terms of the locations given by Eq. (4) of the radio telescopes are readily obtained from Eqs. (4) and (5).

The transformation matrix needed by the Kalman filter is not the coordinate transformation matrix defined by Eq. (6), but rather the transformation matrix that relates the usual UTPM Earth-orientation parameters to the linear combinations of them that are determinable from single-baseline VLBI observations. As defined here, these two determinable components are the transverse  $T$  component representing a right-handed rotation of the Earth about the  $\hat{\mathbf{v}}$ -coordinate direction (which perturbs  $\hat{\mathbf{b}}$  in the positive  $\hat{\tau}$  direction), and the vertical  $V$  component representing a left-handed rotation of the Earth about the  $\hat{\tau}$ -coordinate direction (which perturbs  $\hat{\mathbf{b}}$  in the positive  $\hat{\mathbf{v}}$  direction). The third independent component (not determinable from single-baseline VLBI observations) is the degenerate  $D$  com-

ponent which, as defined here, represents a left-handed rotation about the  $\hat{\mathbf{b}}$ -coordinate direction (that is, about the baseline, thereby perturbing  $\hat{\tau}$  in the negative  $\hat{\mathbf{v}}$  direction). Note that all of the rotations being discussed here are first-order, small angle rotations of the solid Earth (and hence of the attached terrestrial reference frame) with respect to the celestial reference frame. With these definitions of the TVD Earth orientation parameters ( $T, V, D$ ), and remembering that, e.g., a positive change  $\Delta x_p$  in the  $x$  component of the pole position is equivalent to a left-handed rotation of the body-fixed, terrestrial reference frame about the  $\hat{\mathbf{j}}$ -axis, it is straightforward (albeit tedious) to show that the desired UTPM parameter space transformation matrix relating the usual UTPM parameters (PMX, PMY, UT1) to the TVD parameters is given by:

$$\begin{pmatrix} T(t) \\ V(t) \\ D(t) \end{pmatrix} = \begin{pmatrix} -v_y & -v_x & v_z \\ \tau_y & \tau_x & -\tau_z \\ b_y & b_x & -b_z \end{pmatrix} \begin{pmatrix} x_p(t) \\ y_p(t) \\ U(t) \end{pmatrix} \quad (7)$$

where the  $\tau_i, v_i$ , and  $b_i$  elements of the UTPM parameter space transformation matrix in Eq. (7) are the same as those elements of the coordinate transformation matrix in Eq.(6). Note that as with the VUD parameters, the Earth-orientation parameters in Eq. (7) are also all assumed to have the same units.

Apart from a sign difference (due to  $D$  being defined here as a left-handed rotation), the degenerate component of the Earth's orientation which is not determinable from single-baseline VLBI observations is related to the degenerate component which is not determinable from single-station LLR observations. To show this relationship, Eqs.(4) and (5) can be used to write the unit coordinate vector  $\hat{\mathbf{b}}$  in the baseline length direction as:

$$\hat{\mathbf{b}} = \cos \phi \cos \lambda \hat{\mathbf{i}} + \cos \phi \sin \lambda \hat{\mathbf{j}} + \sin \phi \hat{\mathbf{k}} \quad (8)$$

where the angles  $\phi$  and  $\lambda$  are defined by:

$$\phi = \sin^{-1}(b_z) = \sin^{-1} \left( \frac{z_2 - z_1}{\|\mathbf{r}_2 - \mathbf{r}_1\|} \right) \quad (9a)$$

$$\lambda = \tan^{-1}(b_y/b_x) = \tan^{-1} \left( \frac{y_2 - y_1}{x_2 - x_1} \right) \quad (9b)$$

By Eq.(7), the degenerate component of the Earth's orientation not determinable from single-baseline VLBI observations can therefore be written as:

$$D(t) = x_p(t) \cos \phi \sin \lambda + y_p(t) \cos \phi \cos \lambda - U(t) \sin \phi \quad (10)$$

which, by Eq.(1c) and apart from a sign difference, is formally equivalent to the expression for the degenerate component of the Earth's orientation not determinable from single-station LLR observations.

The angles  $\phi$  and  $\lambda$  defined by Eqs.(8) and (9) specify the latitude and east longitude, respectively, of a vector parallel to the baseline, but whose tail is located at the origin of the terrestrial reference frame. Thus, in the case of single-baseline VLBI, the angles  $\phi$  and  $\lambda$  in Eq.(10)

specify the orientation of the baseline vector, whereas in the case of single-station LLR, the angles  $\phi_i$  and  $\lambda_i$  in Eq.(1c) specify the orientation of the station's position vector. If the VLBI baseline and the LLR station position vectors are parallel, then single-baseline VLBI observations and single-station LLR observations will have their degeneracies in the same direction of UTPM parameter space since their indeterminable components result from rotations of the Earth about parallel axes. In the case of single-baseline VLBI, the indeterminable component of the Earth's orientation results from rotations of the Earth about the baseline vector (whose tail can be considered to be located at the origin of the terrestrial reference frame), whereas in single-station LLR the indeterminable component results from rotations about the station's position vector (whose tail is located at the origin of the terrestrial reference frame). In this sense, the baseline vector in single-baseline VLBI plays the same role as does the station position vector in single-station LLR.

#### 2.4 Global positioning system

The GPS has two major elements: (1) a space-based element consisting of a constellation of 24 satellites, and (2) a ground-based element consisting of a network of receivers. The satellites are at altitudes of 20 000 km in orbits of 12-h period and are located equidistant from each other in three orbital planes each inclined at  $55^\circ$  to the Earth's equator. Navigation signals are broadcast by the satellites at two L-band frequencies, thereby enabling first-order corrections to be made for ionospheric refraction effects. The ground-based multichannel receivers detect the navigation signals being broadcast by those satellites that are above the horizon (up to the number of channels in the receiver). A variety of geophysical parameters, including the position of each receiver, and by extension the orientation of the network of receivers as a whole, can be determined by analyzing the detected broadcast signals (e.g., Bock and Leppard 1990; Blewitt 1993; Hofmann-Wellenhof 1993; Beutler et al. 1996).

A number of organizations, including groups at the Astronomical Institute of the University of Bern, the Geodetic Survey of Canada, the Scripps Institution of Oceanography (SIO), and the JPL, currently determine Earth-orientation parameters from GPS measurements. Since these groups use the same basic set of GPS measurements, their results are not completely independent of each other, even though their subsequent data editing and processing procedures are different. For the purpose of combining Earth-orientation series, it is desirable to combine only completely independent determinations of the Earth's orientation. Thus, in generating SPACE95, three nonoverlapping GPS Earth-orientation series were used, two determined at JPL and one at SIO. The first JPL GPS series chosen is designated EOP(JPL) 95 P 02 (Heflin et al. 1995) and consists of daily determinations of polar motion spanning 1 June 1992 to 27 January 1995. A second JPL series, spanning

28 January 1995 to 10 February 1996 and consisting of their routine International GPS Service for Geodynamics (IGS) polar motion determinations (Zumberge et al. 1995), was used to update the first series (although the two series were treated separately in the SPACE95 combination procedure to be described). The particular SIO GPS polar motion series used is that portion of EOP(SIO) 93 P 01 (Bock et al. 1993) that does not overlap with the JPL series, namely, that portion spanning 25 August 1991 through 31 May 1992. Note that because the Kalman filter used at JPL for combining Earth-orientation series does not currently recognize length of day (LOD) as an input data type, only the GPS polar motion determinations were used in generating SPACE95. The incorporation of LOD measurements into the Kalman Earth-orientation filter is currently under study.

### 3 Kalman Earth-orientation filter

Kalman filters are commonly used for estimating parameters of some system when a stochastic model of the system is available and when the data contain noise (e.g., Nahi 1969; Gelb 1974; Bierman 1977). For the purpose of combining Earth-orientation series, the system consists of a series of the usual UTPM parameters, their excitations, and full covariance matrices. The data consist of series of observed Earth-orientation parameters, which may be incomplete and/or degenerate, along with the data measurement covariance matrices. Hernquist et al. (1984) and Eubanks et al. (1985a) have described a Kalman filter and smoother developed at JPL for use with UT1 measurements. The approach currently taken at JPL evolved from the earlier UT1 Kalman filter, and now estimates all three UTPM parameters, along with their excitations, wherein the polar motion parameters are treated as individual real-valued quantities, rather than as a single complex-valued quantity (Morabito et al. 1988). Other approaches using Kalman filters to estimate the polar motion excitation functions (e.g., Barnes et al. 1983) from polar motion measurements have been described by Brzezinski (1990, 1992, 1994) and Preisig (1992).

The particular design of the current JPL Kalman Earth-orientation filter (KEOF) was dictated to a large extent by the nature of the Earth-orientation series being combined, and in particular by the presence of degenerate data types (single-station LLR and single-baseline VLBI observations). This section describes aspects of KEOF that are driven by the characteristics of the input space-geodetic data sets. A brief description of the stochastic models employed in KEOF may be found in Morabito et al. (1988); see also Eubanks et al. (1985b). No discussion is presented here concerning the ability of KEOF to recognize atmospheric angular momentum (AAM) data, which is used in KEOF as a proxy length-of-day measurement, since the emphasis of this report is on combining space-geodetic Earth-orientation series. The incorporation of AAM data into KEOF is described by Freedman et al. (1994).



The incorporation into KEOF of observed data series that consist of all three UTPM parameters (PMX, PMY, UT1) is straightforward. The  $3 \times 1$  measurement vector  $\mathbf{x}_m$  is simply filled with all three observed quantities, and the  $3 \times 3$  measurement error covariance matrix  $\mathbf{C}_m$  is filled with the measurement variances (the square of the measurement uncertainties) and, when correlations are available, the measurement covariances. When the correlations are not available the off-diagonal elements of the measurement covariance matrix are filled with zero. When only a subset of the three usual UTPM parameters is available, as is the case with the IRIS “Intensive” series consisting of only UT1, or with the SLR series wherein the UT1 values have been discarded, then only that subset of the UTPM parameters is used to fill the relevant elements of  $\mathbf{x}_m$  and  $\mathbf{C}_m$ . The other entries of the measurement vector and covariance matrix are filled with zero (actually, any value could be used here, since, as will be discussed, KEOF never accesses those elements of  $\mathbf{x}_m$  and  $\mathbf{C}_m$ ).

Complications arise when the observed quantity is some linear combination of the usual UTPM parameters. In this case, the measurement vector and measurement error covariance matrix are filled in the natural frame of that data type, and then rotated to the usual UTPM frame. (Prior to combination, the necessary adjustments to the measurements and their uncertainties are applied in the frame that is the most convenient one to use for that particular adjustment – the natural frame for the uncertainty adjustment factors and the UTPM frame for all other adjustments including the bias-rate corrections and the removal of leap seconds and tidal effects.) The use of the word “frame” here is not meant to denote a coordinate reference frame, but rather different sets of independent, linear combinations of the Earth-orientation parameters spanning the UTPM parameter space, such as the VUD and TVD sets. In the case of single-station LLR observations, the observed  $UT0_i - TAI$  values (and their uncertainties) are first converted to  $UTF_i$  values (and their uncertainties) by Eq.(2). Nominal values for the locations of the LLR observing stations are used to evaluate the transformation matrix defined by Eq.(3), which is then used to transform the VUD parameters and associated error covariance matrix back to the usual UTPM frame. In doing this, zeros are used to fill those elements of the measurement vector and error covariance matrix in the VUD frame corresponding to the degenerate component of the Earth’s orientation not determinable from LLR observations taken at that single station. In the case of single-baseline VLBI observations, the measurement vector and associated measurement error covariance matrix are filled in the TVD frame, with zeros being used to fill those elements of the measurement vector and error covariance matrix corresponding to the degenerate component of the Earth’s orientation that is not determinable from VLBI observations taken on that single baseline. The transformation matrix defined by Eq.(7) is then evaluated using nominal values for the locations of the two radio telescopes defining the baseline. This transformation matrix is then used to trans-

form the TVD parameters and associated error covariance matrix back to the usual UTPM frame.

The use of zeros to fill the elements of the measurement error covariance matrix corresponding to the degenerate component is not meant to indicate that the degenerate component is perfectly known. In fact, in principle, the degenerate component is perfectly unknown, and therefore of infinite variance. However, for numerical reasons, these elements of the measurement error covariance matrix must be filled with some value other than infinity, and zero was chosen for this purpose. The fact that the measurement error covariance matrix associated with degenerate data types cannot be properly represented numerically has required that KEOF be formulated to use the information matrix, rather than the covariance matrix.

The measurement information matrix  $\mathbf{C}_m^{-1}$  is the inverse of the measurement error covariance matrix. Elements of the information matrix corresponding to the degenerate component are properly filled with zero, indicating that the observations contain no information about the degenerate direction in UTPM parameter space. The measurement error information matrix is generated by inverting that submatrix of the measurement error covariance matrix corresponding to the observed components (after rotating back to the natural reference frame for that data type, if necessary). The elements of the information matrix corresponding to the degenerate (or, in some cases, discarded) components are (now properly) set to zero. The resulting full  $3 \times 3$  information matrix associated with degenerate data types will be rank deficient. However, the eigenvectors associated with the zero eigenvalues span the degenerate subspace of UTPM parameter space that is not determinable from observations by that data type (or that have been purposely discarded). Thus, even in the usual UTPM frame, the information matrix is always appropriately defined, even for the degenerate data types.

The Kalman filter contains a stochastic model of the process which is used to propagate the state vector (and its associated state covariance matrix) forward to the time of a measurement. The state vector  $\mathbf{x}_s$  contains values for the parameters being estimated by the Kalman filter. In the case of KEOF, the  $11 \times 1$  state vector  $\mathbf{x}_s$  consists of the UTPM parameters (PMX, PMY, UT1) and their excitations, along with additional parameters associated with the model of the polar motion excitation and the use of AAM data. At the time  $t$  of a measurement, both an initial estimate  ${}_i\mathbf{x}_s(t)$  of the state vector [along with its associated  $11 \times 11$  error covariance matrix  ${}_i\mathbf{C}_s(t)$ ] and the  $3 \times 1$  measurement vector  $\mathbf{x}_m(t)$  [along with its associated  $3 \times 3$  measurement information matrix  $\mathbf{C}_m^{-1}(t)$ ] are available. The state vector is updated at the measurement time  $t$  by forming the vector weighted average of its initial estimate with the measurement vector:

$$\mathbf{x}_s(t) = [{}_i\mathbf{C}_s^{-1}(t) + \mathbf{H}^T \mathbf{C}_m^{-1}(t) \mathbf{H}]^{-1} \times [{}_i\mathbf{C}_s^{-1}(t) {}_i\mathbf{x}_s(t) + \mathbf{H}^T \mathbf{C}_m^{-1}(t) \mathbf{x}_m(t)] \quad (11)$$



where  $\mathbf{H}$  is a  $3 \times 11$  matrix which relates the elements of the state vector to the elements of the measurement vector:

$$\mathbf{x}_m(t) = \mathbf{H} \mathbf{x}_s(t) + \mathbf{n}_m(t) \quad (12)$$

with  $\mathbf{n}_m(t)$  being a  $3 \times 1$  vector representing the measurement noise that is assumed to be white, Gaussian, of zero expectation, and uncorrelated with the stochastic process noise. Since Eq.(11) is formulated in terms of information matrices, it is valid and can be used even in the presence of degenerate data types. The state covariance matrix is updated at the measurement time  $t$  by inverting the sum of the initial state and measurement information matrices:

$$\mathbf{C}_s(t) = [\mathbf{I} \mathbf{C}_s^{-1}(t) + \mathbf{H}^T \mathbf{C}_m^{-1}(t) \mathbf{H}]^{-1} \quad (13)$$

Smoothed estimates of the Earth-orientation parameters are obtained by running the Kalman filter forward in time, backward in time (e.g., Gelb 1974), and taking the vector weighted average of the results. Thus, the final output of the Kalman filter consists of a series of smoothed, interpolated estimates of the usual UTPM parameters (PMX, PMY, UT1), their excitations, and full covariance matrix that is based upon independent measurements, whether or not they are incomplete, degenerate, or irregularly spaced in time.

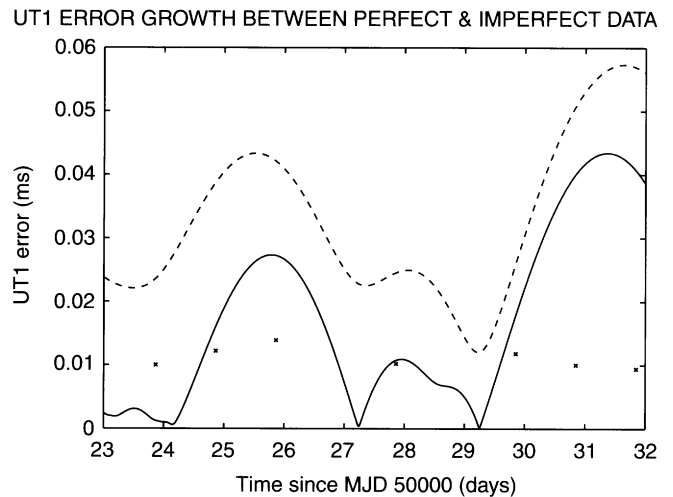
#### 4 An approach to combining Earth-orientation series

A Kalman filter has a number of useful properties that make it an attractive choice as a means of combining independent Earth-orientation data sets. Changes in the Earth's orientation can be described as a randomly excited stochastic process. Consequently, between successive measurements of the Earth-orientation parameters, the uncertainty in the knowledge of their values grows and rapidly becomes much larger than the uncertainty in the measurements. Thus, it is important to analyze each measurement at its measurement epoch, rather than at some "nearby", regularized epoch, as is commonly done in normal-point methods of combining data sets. Kalman filters are an effective means of dealing with irregularly spaced data sets, since the state vector and state covariance matrix can be propagated to the measurement epoch regardless of whether or not the measurements are equispaced.

Due to this growth in the uncertainty of the Earth-orientation parameters between measurements, when intercomparing data sets in order to evaluate their relative accuracies (and hence to set the uncertainty adjustment scale factors), it is important to compare independent measurements whose epochs are as close as possible to each other. This argues for the comparison of an individual data set against a combination of all other independent data sets (rather than against some other individual series), so that the difference in the epochs of the measurements being compared is minimized. Since it is unlikely that independent measurements will be given at exactly the same epoch, it is

important that the interpolation procedure used in generating the combined series accounts for the growth in the uncertainty of the Earth-orientation parameters between measurements. The JPL KEOF does this in an objective manner (see previous section) by employing realistic stochastic models of the uncertainty growth between measurements (Morabito et al. 1988).

The importance of accounting for interpolation error between measurements is illustrated in Fig. 2, which shows the uncertainty in the UT1 component of two different USNO IRIS "Intensive" complementary smoothings formed by combining all but the "Intensive" series used in forming SPACE95 (see below). In generating the solid line the uncertainties of the measurements have been reduced by a factor of  $10^4$  prior to combination in order to model the error growth between perfectly known measurements, whereas the dashed line was generated without modification of the measurement uncertainties. With perfectly known measurements the UT1 uncertainty of the complementary smoothing depends only on the error in interpolating between the measurements, whereas with imperfect measurements, the UT1 uncertainty at any time is a combination of interpolation and measurement error. The relative importance of these two sources of UT1 uncertainty can be assessed by comparing the solid and dashed lines. As can be seen, for example near MJD 50026 and MJD 50031, the UT1 uncertainty is often larger than the difference between the two curves, indicating that between measurements the uncertainty in the UT1 component of combinations of Earth-orientation series is often dominated by interpolation, rather than measurement, error. The crosses in Fig. 2 indicate the uncertainties and epochs of the USNO IRIS "Intensive"



**Fig. 2.** Illustration of UT1 error growth between measurements in two different USNO IRIS "Intensive" complementary smoothings. The *solid line* was generated by reducing the measurement uncertainties by a factor of  $10^4$  prior to combination in order to model the error growth between perfect measurements; the *dashed line* was generated without modification of the measurement uncertainties. The *crosses* indicate the epochs and adjusted (by the scale factor given in Table 7) uncertainties of the USNO IRIS "Intensive" measurements taken during this time-interval

measurements taken during the 9-day-long time-interval illustrated (note that the stated uncertainties of the “Intensive” measurements have been adjusted here by the factor given in Table 7). The “Intensive” measurements are seen generally to occur between the epochs of the measurements in the complementary smoothings. Thus, the “Intensive” measurements frequently occur at times when interpolation error dominates the UT1 uncertainty in the complementary smoothings. Therefore, accounting for interpolation error, and not just measurement error, is important when combining measurements, since this is often the dominant source of uncertainty. This is particularly important when evaluating the measurement uncertainty of one series by intercomparison with other series. Kalman filters, by virtue of incorporating stochastic models of the process and measurement noise, are particularly well suited to this task.

Finally, in order to obtain the best possible combined series, the degree of smoothing applied to the measurements must vary with both the precision and the sampling interval of the measurements. As improvements have been made to the measurement systems (in both hardware and software), the precision with which the measurements have been made has dramatically improved, as has their time resolution. With a Kalman filter, the degree of smoothing applied [see Eq.(11)] is a function of both the precision of the measurements and the time-span over which the state vector and covariance matrix must be propagated (i.e., of the time resolution of the measurements). (Of course, the Kalman filter model for the growth in the uncertainty of the Earth-orientation parameters between measurements is also important in this regard.) Thus, with a Kalman filter, no arbitrary changes in the applied degree of smoothing need be made. The degree of smoothing is automatically adjusted as the precision and time resolution of the measurements changes.

Because of these considerations, a Kalman-filter-based approach to combining Earth-orientation series has been taken at JPL. But prior to combining independent estimates of the Earth’s orientation, a number of corrections must be applied to the individual series. First, the series are likely to exhibit differences in bias and rate due to differences in the underlying reference frames within which they are given. A conventional terrestrial reference frame is realized in practice by specifying the positions and secular motions of a set of observing stations that are globally distributed on the surface of the Earth. When deriving the Earth-orientation series, the locations and velocities of the observing sites must be specified, and are, in fact, usually estimated simultaneously with the UTPM parameters. Thus, in principle, each solution for the Earth-orientation parameters is given within its own reference frame. Constraints are typically applied during the data reduction procedure in order to place a given solution within a particular reference frame, but different constraints are employed by the various analysis centers leading to differences in the underlying reference frames, and hence to differences in the bias and rate of the determined

Earth-orientation series. Also, the observing stations of the various techniques are located on different subsets of the set of mobile, deformable tectonic plates. A model of the plate motions is usually employed when reducing Earth-orientation observations, but discrepancies between the plate model and the true tectonic motions of the stations as evidenced by their adjusted velocities can lead to differences in the rates of the Earth-orientation series, especially when the observing stations are located on different subsets of the plates. Thus, Earth-orientation series derived by different techniques, or from independent observations by the same technique but using different station sets, can be expected, in principle, to exhibit differences in bias and rate, and therefore provision must be made to determine and apply bias-rate corrections prior to their combination.

Each of the observed Earth-orientation series is a measure of the Earth’s true orientation. Since the true orientation of the Earth is not known, it is impossible to know which of the observed series is the most accurate measure of the Earth’s orientation. The stated uncertainties distributed with each data set indicate the internal precision with which the observations have been made, but do not necessarily reflect the accuracy of those observations. An estimate of the relative accuracy of the different data sets can be obtained through their intercomparison. (Of course, the true accuracy of the data sets cannot be obtained through intercomparison studies due to the possible presence of systematic errors common to all the data sets.) In the absence of a priori knowledge about the relative accuracy of the different data sets, at the beginning of the intercomparison process they should all be treated as though they are potentially equally accurate, but, of course, not equally precise.

In generating SPACE95, no reference series has been used for the purpose of determining the bias and rate corrections that must be applied to each series in order for them to be consistent with each other (in bias and rate) prior to their combination. First, none of the individual series listed in Table 1 are appropriate choices for a reference series, since none have the requisite time resolution, span a great enough time-interval, or consist of the full set of Earth orientation parameters. Also, any available combined Earth-orientation series is a dubious choice for a reference series, since it may contain spurious variations due to inadequacies of the combination technique, thereby affecting the desired bias-rate corrections. Finally, a democratic treatment of the individual data sets precludes the use of a reference series. Inherent in the use of a reference series is the belief that the particular series chosen as the reference more accurately represents the true orientation of the Earth than do any of the other available series. Since, as already discussed, the true accuracy of any observed series cannot be known, the decision about which series to use as a reference is, to some extent, a subjective one. In generating SPACE95, care has been taken to make the process be as free as possible from the need to make such subjective decisions, and, therefore, no reference series has been used. Rather, for the purpose of deter-

mining the bias-rate corrections, each data set has been iteratively compared to a combination of all other data sets. Thus, all data sets are treated equally, with no subjective decision being made about their presumed accuracies.

For SPACE95, the bias-rate corrections have been determined by an iterative scheme wherein, during a given iteration, each individual series is compared to a combination of all other series (this combination of all other series is hereafter called the complementary smoothing for that particular individual series). Ideally, in order to minimize interpolation error when subsequently forming the residual series (see below), the complementary smoothing should be produced by a Kalman filter that interpolates to and prints its estimates at the exact measurement epochs of the corresponding individual series. However, due to software limitations, this ideal case is currently approximated by using the Kalman filter to produce a regular smoothing with output printed at 1-day intervals and then linearly interpolating the output values and covariance matrices to the exact measurement epochs of the corresponding individual series.

Within each iteration, each series is analyzed in parallel with the analyses of each of the other series (i.e., the order in which each series is analyzed is completely immaterial in this procedure). Half the incremental bias-rate corrections determined during a given iteration are applied to the series and the process repeated until convergence is attained, convergence being indicated by the incremental bias-rate corrections approaching zero. The reason for applying only half the bias-rate corrections determined during any iteration can be understood by considering the application of this procedure to just two series, labeled A and B. By following this procedure, bias-rate corrections would be determined that make the bias and rate of series A agree with that of B, and vice versa. Upon applying the full corrections thus determined, the corrected series A would have the bias and rate of the original series B, and the corrected series B would have the bias and rate of the original series A. Thus, the bias-rate difference between series A and B after correction would be the same size as that before correction. This nonconverging situation can be avoided if only half the corrections that were determined are actually applied to each series. In this case, the corrected series will then agree with each other in bias and rate.

The bias-rate corrections are determined by a weighted least-squares fit of a bias and rate to each individual Earth-orientation component of the residual series, with the weights being based upon the uncertainties of the residuals, and with the residual series being analyzed in the natural Earth-orientation reference frame of the individual series. The residual series is formed by differencing the given individual series with a combination of all other series (the complementary smoothing) after both the individual and combined series have been transformed (if necessary) to the natural Earth-orientation reference frame of the individual series (that is, the difference is formed in the natural ref-

erence frame of the individual series). Since the complementary smoothing is independent of the individual series, the covariance matrices associated with the residual values are formed by simply summing the covariance matrices of the individual series with those of the complementary smoothing, after the covariance matrices have also been transformed (if necessary) to the natural reference frame of the individual series. Since the bias-rate corrections are determined separately for each component, only the diagonal elements of the residual covariance matrices corresponding to that component being corrected are used in the weighted least-squares fit. This single component (or single variate) approach to analyzing the residual series results from current software limitations that preclude the use of a multivariate approach. Future software upgrades will allow the use of a multivariate approach in which all components of the residual series are analyzed in unison, thereby allowing correlations between components to be taken into account in the analysis.

Along with determining bias-rate corrections, the stated uncertainties in each series are also adjusted during the iterative procedure by determining a scale factor which, when applied to those uncertainties, makes the residual of that series upon differencing it with a combination of all other series have a reduced chi-square of 1. Note that by means of the Kalman filter's stochastic model of the Earth rotation process, the Kalman filter's estimate of the covariance matrix of the complementary smoothing at each epoch accounts for the interpolation error at that epoch due to the stochastic character of the Earth rotation process. Of course, the covariance matrix of the complementary smoothing also depends upon the measurement uncertainties of the series being combined in the complementary smoothing. Thus, the covariance matrix of the residual, being the sum of the covariance matrices of the individual series and its complementary smoothing, accounts for the uncertainty of the interpolation as well as the measurement uncertainties of all the data series. The purpose of adjusting the stated uncertainties of the observations is to make them consistent with the scatter in the residual series. Such consistent estimates for the uncertainties of the observed values are important because the Kalman filter uses the uncertainty estimates as weights when combining the observations with the initial-state estimate [see Eq.(11)].

The iterative procedure to determine bias-rate corrections and uncertainty adjustment factors is, in fact, done primarily for the purpose of determining the uncertainty adjustment factors, not the bias-rate corrections. The bias-rate corrections could be more easily determined by other means, such as by estimating them within the Kalman filter when the Earth-orientation series are combined. However, the uncertainty adjustment factors cannot be estimated in this way by the Kalman filter and must be determined and applied a priori. The iterative procedure being described here was developed primarily for this purpose, although it also yields estimates of the required bias-rate corrections.

Since the uncertainties are being adjusted along with the bias and rate of the series, and since the bias-rate corrections are determined by a weighted least-squares fit with the weights being dependent upon the values of the adjusted uncertainties, it is important that during each iteration the uncertainty scale factors and bias-rate corrections are determined simultaneously. The uncertainty scale factor is determined separately for each component by analyzing the scatter in the fit of a bias and rate to that component of the residual series, with the residual series being formed and the bias-rate fit being computed as described. Once that value for the uncertainty scale factor has been found which upon application leads to a value of 1 for the reduced chi-square in the scatter about the fit for a bias and rate to the residual series, then half those particular values for the bias and rate are taken as the desired bias-rate corrections that are to be applied to the individual series, and that value for the uncertainty scale factor is taken as the desired scale factor to be applied to the uncertainties.

During the iterative procedure for the determination of bias-rate corrections and uncertainty scale factors, individual data points are deleted whose residual values are greater than three times their adjusted uncertainties (the residual values being obtained from the difference of that data set with its complementary smoothing). Since the uncertainties are being adjusted during the iterative procedure, a sufficient number of iterations must first be completed in order to converge upon stable values for the adjusted uncertainties (convergence here being indicated by the incremental uncertainty scale factors approaching 1). During subsequent iterations, outlying data points can then be deleted. Currently, outliers are identified by separately examining each component of the residual series, although in the future outliers will be identified using a multivariate approach. All components (e.g. PMX, PMY, and UT1) of the data point are deleted even if only one component is the outlier, and once a data point has been deleted it remains deleted for all subsequent iterations. Of course, during subsequent iterations, the incremental uncertainty scale factors determined for some data set that has had outliers deleted during previous iterations are usually found to be such as to reduce the adjusted uncertainties, thereby usually causing additional data points to be considered outliers and to be deleted. After a sufficient number of iterations, however, the incremental scale factors will converge to 1, and no more data points will be found to have residual values greater than three times their adjusted (and converged) uncertainties.

## 5 SPACE95

Summary information about the particular data sets combined to form SPACE95 is given in Table 1. Prior to their combination, corrections to the bias, rate, and stated uncertainties of each series were determined (and applied) by the preceding iterative approach (except for

the GPS and USNO series which were treated separately, as will be described). But first, leap seconds and the effect of the solid Earth and ocean tides on the Earth's rotation (UT1) were removed. The model of Herring (1993) – see also Herring and Dong (1994) – was used to remove the effect of the diurnal and semidiurnal ocean tides from those UT1 observations that included them in their reported values, namely, the NOAA IRIS “Intensive” series. The model of Yoder et al. (1981) was used to remove the effect on UT1 of the solid Earth tides between the lunar nodal (18.6 years) and fortnightly periods, with the model of Dickman (1993) being used to remove the effect on UT1 of the  $Mf$ ,  $Mf'$ ,  $Mm$ , and  $Ssa$  ocean tides in this long-period tidal band. Since the Yoder et al. (1981) model already includes a contribution from the equilibrium ocean tides, just the values given in Table 2 for the Dickman (1993) oceanic tidal corrections to the Yoder et al. (1981) model were actually employed when removing the effect on UT1 of the long-period solid Earth and ocean tides. The values given in Table 2 for the oceanic tidal corrections were obtained by first computing the tidal effect on UT1 of an oceanless-Earth model by multiplying the negative of the tidal amplitudes given in Table 1 of Yoder et al. (1981) by  $k/C = 0.8073$  (Williams, personal communication, 1992). To this were added the model results of Dickman (1993) for the effect on UT1 of the long-period ocean tides. The standard model of Yoder et al. (1981), formed by multiplying the negative of the tidal amplitudes given in their Table 1 by  $k/C = 0.94$ , was then subtracted from this total effect, thereby obtaining the entries given here in Table 2 for the Dickman (1993) oceanic tidal corrections to the Yoder et al. (1981) model.

In some cases, such as for the Bureau International de l'Heure (BIH) optical astrometric series used in generating COMB94 (Gross 1996a), the observations span a large enough fraction of the tidal period that the tidal amplitude should be attenuated prior to being removed from the UT1 observations. The attenuation factor that should be applied to the amplitude of each of the tidal terms is a function of the tidal frequency as well as of the length of the observation window (e.g., Guinot 1970). Consider a tidally induced UT1 signal of amplitude  $A$ , frequency  $\omega_0$ , and phase  $\alpha$ :

$$U(t) = A \cos(\omega_0 t - \alpha) \quad (14)$$

Observations of duration  $T$  centered at epoch  $t_0$  effectively average this signal by:

**Table 2.** Applied oceanic corrections to the solid Earth tide model

tide	fundamental argument					period (solar days)	correction	
	$l$	$l'$	$F$	$D$	$\Omega$		sine ( $\mu$ s)	cosine ( $\mu$ s)
$Mf'$	0	0	2	0	1	13.63	0.14	8.58
$Mf$	0	0	2	0	2	13.66	0.23	20.73
$Mm$	1	0	0	0	0	27.55	−9.13	12.09
$Ssa$	0	0	2	−2	2	182.62	−75.21	11.07

$$\begin{aligned}
\langle U(t) \rangle &\equiv \frac{\int_{t_0-T/2}^{t_0+T/2} A \cos(\omega_0 t - \alpha) dt}{\int_{t_0-T/2}^{t_0+T/2} dt} \\
&= \frac{\sin(\frac{1}{2}\omega_0 T)}{\frac{1}{2}\omega_0 T} A \cos(\omega_0 t_0 - \alpha)
\end{aligned} \quad (15)$$

where the angular brackets  $\langle \rangle$  denote the averaging operation. Thus, the amplitude attenuation factor to be applied is  $\sin(\omega_0 T/2)/(\omega_0 T/2)$ .

After removal of leap seconds and tidal terms (both solid Earth and oceanic) from the UT1 observations, bias-rate corrections and uncertainty scale factors were determined for each series by the iterative procedure described in the previous section. In an attempt to reduce the required number of iterations, initial values of the bias-rate corrections (but not for the uncertainty scale factors) were obtained by the use of a reference series – SPACE94 (Gross 1995, 1996a). Any inconsistencies introduced by use of a reference series for this initial bias-rate correction should be eliminated during the subsequent iterative procedure. The initial bias-rate corrections needed to align each series with the reference series are given in Table 3. The uncertainties of these initial bias-rate corrections (given in parentheses in the table) are the  $1\sigma$  standard errors in their determination.

No bias-rate corrections are listed in Table 3 for those Earth-orientation parameters that were either discarded (such as the UTC SR SLR UT1 values) or not available (such as the PMX and PMY values of the NOAA IRIS “Intensive” series). Also, no rate corrections are given in Table 3 for those series (such as the SGP/CDP Westford–Mojave single baseline VLBI series) spanning such a short time-interval that reliable rate corrections could not be determined for them.

After initial bias-rate alignment, final values for the bias-rate corrections and uncertainty scale factors were determined by the iterative procedure described in the previous section. During this iterative procedure, each data type was analyzed and corrections determined in its own natural reference frame. For single-station LLR observations this is the VUD frame defined by the location of that particular station at which the observations are made, and for single-baseline VLBI observations it is the TVD frame defined by the orientation of that particular baseline over which the observations are made. At each stage in this iterative procedure, every residual series was visually examined in order to check for the possible presence of systematic errors or problems with the procedure (none were found). Outlying data points (those whose residual values are greater than three times their adjusted uncertainties) were rejected during this process, but not until four iterations had been completed in order to converge

**Table 3.** Initial adjustments made prior to iterative procedure

data set name	bias (mas)		rate (mas/year)			
LLR (JPL95M01)	VOL		UT0	VOL		UT0
McDonald cluster	–1.211 (0.127)		0.085 (0.137)	–0.741 (0.024)		–0.154 (0.024)
CERGA	0.494 (0.039)		–0.047 (0.040)	0.147 (0.020)		–0.006 (0.016)
Haleakala	–0.413 (0.578)		–0.875 (0.438)	–0.290 (0.117)		–0.095 (0.094)
DSN (JPL95R01)	T		V	T		V
California–Spain Cluster	0.202 (0.022)		0.019 (0.061)	0.086 (0.010)		0.011 (0.028)
California–Australia Cluster	0.331 (0.017)		–0.128 (0.058)	–0.078 (0.007)		–0.059 (0.024)
SGP/CDP (GLB973f)	T		V	T		V
Westford–Ft. Davis	8.188 (0.841)		1.793 (4.761)	0.714 (0.083)		2.174 (0.477)
Westford–Mojave	0.498 (0.109)		0.623 (0.432)	–		–
SGP/CDP (GLB973f)	PMX	PMY	UT1	PMX	PMY	UT1
Multibaseline	–1.149 (0.007)	–1.992 (0.006)	0.469 (0.007)	–0.141 (0.003)	–0.059 (0.003)	–0.117 (0.003)
NOAA (NOAA95R02)	PMX	PMY	UT1	PMX	PMY	UT1
IRIS Intensive	–	–	0.729 (0.021)	–	–	–0.077 (0.006)
UTCSR (CSR95L01)	PMX	PMY	UT1	PMX	PMY	UT1
Lageos	0.027 (0.020)	0.064 (0.018)	–	0.108 (0.006)	–0.042 (0.005)	–

Reference date for bias-rate adjustment is 1993.0

upon initial estimates for the uncertainty scale factors. A further five iterations were required to converge on final bias-rate corrections and uncertainty scale factors. During the final iteration, no data points had residual values greater than three times their adjusted uncertainties, the values for all the incremental bias-rate corrections were less than the standard errors in their determination, and the incremental scale factors changed the uncertainties by much less than 1%. The product of all the uncertainty scale factors determined and applied during the iterative procedure, along with the sum of all the bias-rate corrections applied (half those determined), are shown in Table 4. The uncertainties given in parentheses in Table 4 are the  $1\sigma$  standard errors in the determination of the bias-rate corrections during the last iteration. A total of 253 data points (including the discarded USNO and GPS data points – see below), or about 2% of the available data points, were considered to be outliers and therefore discarded during the procedure to determine bias-rate corrections and uncertainty scale factors.

For the purpose of determining bias-rate corrections and uncertainty scale factors, the LLR stations at McDonald Observatory were clustered, so that a common bias-rate correction and uncertainty scale factor was determined for all the McDonald LLR series. This was done so that a rate determination could be made for them. There is not enough overlap between any indi-

vidual McDonald LLR series and the other, independent series for a reliable rate determination to be made. But by clustering the McDonald LLR stations, there is then enough overlap that a common rate can be determined for all the McDonald LLR series. Similarly, the individual DSN radio telescopes in California were clustered, as were those in Spain, and, separately, in Australia, so that a common bias-rate correction and uncertainty scale factor was determined for all the California–Spain series, as well as for all the California–Australia series.

Neither USNO series was included in the iterative procedure for bias-rate correction and uncertainty-scale-factor determination since there is not enough overlap between their independent portions and the other series for reliable determinations of these corrections to be made (see Table 1). Instead, the bias-rate corrections and uncertainty scale factors for the two USNO series were determined by individually comparing them to two different reference series formed by two different special combinations of the other corrected series. In forming these reference series, the GPS series were not included in either of them (see below), and in order for each reference series to be completely independent of its respective USNO series, only those portions of the SGP/CDP multibaseline and NOAA IRIS “Intensive” series that did not overlap in time with its respective USNO series were selected and included. That is, only that

**Table 4.** Total adjustments made during iterative procedure

data set name	bias (mas)		rate (mas/year)			uncertainty scale factor			
LLR (JPL95M01)	VOL		UT0	VOL	UT0	VOL		UT0	
McDonald Cluster	1.059 (0.140)		–0.004 (0.130)	0.184 (0.041)	–0.009 (0.036)	1.630		1.154	
CERGA	0.133 (0.068)		0.000 (0.055)	0.014 (0.032)	0.015 (0.021)	1.788		1.423	
Haleakala	0.418 (0.915)		–0.643 (0.745)	–0.011 (0.186)	–0.076 (0.157)	1.549		1.692	
DSN (JPL95R01)	T		V	T	V	T		V	
CA–Spain Cluster	–0.071 (0.032)		0.042 (0.074)	0.014 (0.015)	0.053 (0.035)	1.354		1.094	
CA–Australia Cluster	0.051 (0.027)		0.053 (0.071)	0.030 (0.011)	0.000 (0.031)	1.371		1.098	
SGP/CDP (GLB973f)	T		V	T	V	T		V	
Westford–Ft. Davis	3.148 (3.729)		–1.230 (6.555)	0.368 (0.376)	–0.112 (0.657)	0.904		0.870	
Westford–Mojave	–0.233 (0.210)		0.136 (0.439)	–	–	2.326		0.954	
SGP/CDP (973f)	PMX	PMY	UT1	PMX	PMY	UT1	PMX	PMY	UT1
Multibaseline	–0.043 (0.015)	0.011 (0.013)	–0.036 (0.019)	0.014 (0.006)	0.005 (0.005)	0.005 (0.007)	2.226	1.963	2.192
NOAA (95R02)	PMX	PMY	UT1	PMX	PMY	UT1	PMX	PMY	UT1
IRIS Intensive	–	–	–0.027 (0.022)	–	–	0.014 (0.006)	–	–	0.933
UTCSR (95L01)	PMX	PMY	UT1	PMX	PMY	UT1	PMX	PMY	UT1
Lageos	–0.027 (0.020)	0.019 (0.016)	–	0.025 (0.006)	0.010 (0.005)	–	0.849	0.743	–

Reference date for bias-rate adjustment is 1993.0

portion of the SGP/CDP multibaseline series before 1 January 1988 was selected and included (along with the entire NOAA IRIS “Intensive” series) in the reference series used to adjust the USNO multibaseline series; and only that portion of the NOAA IRIS “Intensive” series before 21 December 1993 was selected and included (along with the entire SGP/CDP multibaseline series) in the reference series used to adjust the USNO IRIS “Intensive” series. (In order to be able to determine reliable rate corrections for the USNO multibaseline series, all of its data points after 1 January 1988 were used, even though only those after 29 December 1994 ultimately get incorporated into SPACE95. Similarly, in determining the bias-rate correction for the USNO IRIS “Intensive” series, all of its values (beginning 21 December 1993) were used, even though only those after 4 January 1995 ultimately get incorporated into SPACE95.)

Thus, the two different reference series used to determine the bias-rate corrections and uncertainty scale factors of the two USNO series were formed by combining the respective truncated SGP/CDP multibaseline or NOAA IRIS “Intensive” series with all the other series (but not the GPS series) after the bias-rate corrections and uncertainty scale factors determined in the foregoing for all the other series (Tables 3 and 4) had been applied to them. During the comparison for bias-rate correction and uncertainty-scale-factor determination, outlying data points (those whose residual values were greater than three times their adjusted uncertainties) were also discarded. Table 5 gives the bias-rate corrections and uncertainty scale factors thus determined for the USNO series. The values given in parentheses are the  $1\sigma$  standard errors in the determination of these corrections.

The GPS series were similarly not included in the foregoing iterative procedure for bias-rate correction and uncertainty-scale-factor determination. Rather, bias-rate corrections and uncertainty scale factors for the GPS series were determined by comparing them to combinations of all the other corrected series, including

the USNO series. The bias-rate corrections and uncertainty scale factors determined for all the other series (Tables 3–4, and Table 5 for the USNO series) were applied, the corrected series combined, and the result used as a reference against which the GPS series were compared for the purpose of determining their bias-rate corrections and uncertainty scale factors. No rate corrections were determined for the Scripps and JPL IGS Analysis GPS series, since their overlap with the other, corrected series was not great enough to allow reliable rate corrections to be determined. Table 5 gives the bias-rate corrections and uncertainty scale factors thus determined for the GPS series. The values given in parentheses are the  $1\sigma$  standard errors in the determination of these corrections.

The final step taken prior to generating SPACE95 was to place the corrected Earth-orientation series within a particular IERS reference frame. This was done by applying to them an additional bias-rate correction that is common to all of them. This additional common correction was determined by combining all the series, after applying to them the corrections that have been previously determined for them (Tables 3–5). This combination was then compared to the IERS combination designated EOP(IERS) C 04 (IERS 1995) during the interval 1984–1995 in order to obtain the bias-rate corrections that make it, and hence each of the corrected individual series, have the same overall bias and rate as the IERS series during 1984–1995. The bias-rate corrections thus determined are shown in Table 6, with the  $1\sigma$  standard errors in their determination given in parentheses.

At this point, all the necessary corrections to all the series have been determined. It now remains to apply these corrections and combine the corrected series into SPACE95. Table 1 gives the actual number of data points and their span in time for each series that was used in generating SPACE95. Table 7 gives the corrections made to each of the raw series listed in Table 1 prior to their combination, with the bias-rate entries in Table 7 being the sum of the bias-rate entries in Tables

**Table 5.** Adjustments made to USNO and GPS series

data set name	bias (mas)			rate (mas/year)			uncertainty scale factor		
	PMX	PMY	UT1	PMX	PMY	UT1	PMX	PMY	UT1
USNO (N9604) Multibaseline	–0.143 (0.016)	1.249 (0.012)	–0.751 (0.019)	0.220 (0.009)	0.117 (0.007)	0.052 (0.010)	2.012	1.602	1.783
USNO (N9604.INT) IRIS Intensive	–	–	–0.945 (0.113)	–	–	0.166 (0.059)	–	–	1.840
GPS (SIO93P01) Scripps	–1.081 (0.035)	–1.457 (0.039)	–	–	–	–	1.956	1.903	–
GPS (JPL95P02) JPL	–0.184 (0.025)	–0.200 (0.023)	–	0.084 (0.021)	–0.139 (0.019)	–	3.257	2.800	–
GPS (IGS Analysis) JPL	0.460 (0.035)	0.278 (0.016)	–	–	–	–	12.273	3.542	–

Reference date for bias-rate adjustment is 1993.0



**Table 6.** Common adjustment made to align series with EOP (IERS) C 04

bias (mas)			rate (mas/year)		
PMX	PMY	UT1	PMX	PMY	UT1
−0.010 (0.011)	−0.059 (0.010)	0.046 (0.016)	−0.027 (0.003)	−0.018 (0.002)	−0.009 (0.004)

Reference date for bias-rate adjustment is 1993.0

3–6. Note that the same IERS rate correction is applied to all of the series, including those (such as the Scripps and JPL IGS Analysis GPS series) for which no relative rate correction could be determined. Thus, the rate correction given in Table 7 for those series for which no relative rate correction could be determined is simply the IERS rate correction, but given, of course, in the natural reference frame for that series. The resulting combination, SPACE95, consists of daily values at midnight of PMX, PMY, and UT1, their  $1\sigma$  formal errors and correlations spanning 6.0 October 1976 to 10.0 February 1996. Figure 3 illustrates the resulting UT1 and polar motion values, with Fig. 4 illustrating the  $1\sigma$  formal

errors in the determination of the UTPM values. Leap seconds, and the effect on UT1 of the long-period (fortnightly and longer) solid Earth and ocean tides have been added back to the UT1 values using the same models for these effects that were originally used in removing them from the raw series [Yoder et al. (1981) for the long-period solid Earth tides and Dickman (1993) for the long-period ocean tides]. However, no semidiurnal or diurnal tidal terms have been added back. Over time, improvements to the observing systems (in both hardware and software) have led to increasingly precise determinations of the Earth's orientation. This improvement is reflected in SPACE95 by the reduction of the formal errors from about 2 mas in polar motion and 0.5 ms in UT1 during the late 1970s to their recent values of about 0.2 mas in polar motion and 0.02 ms in UT1.

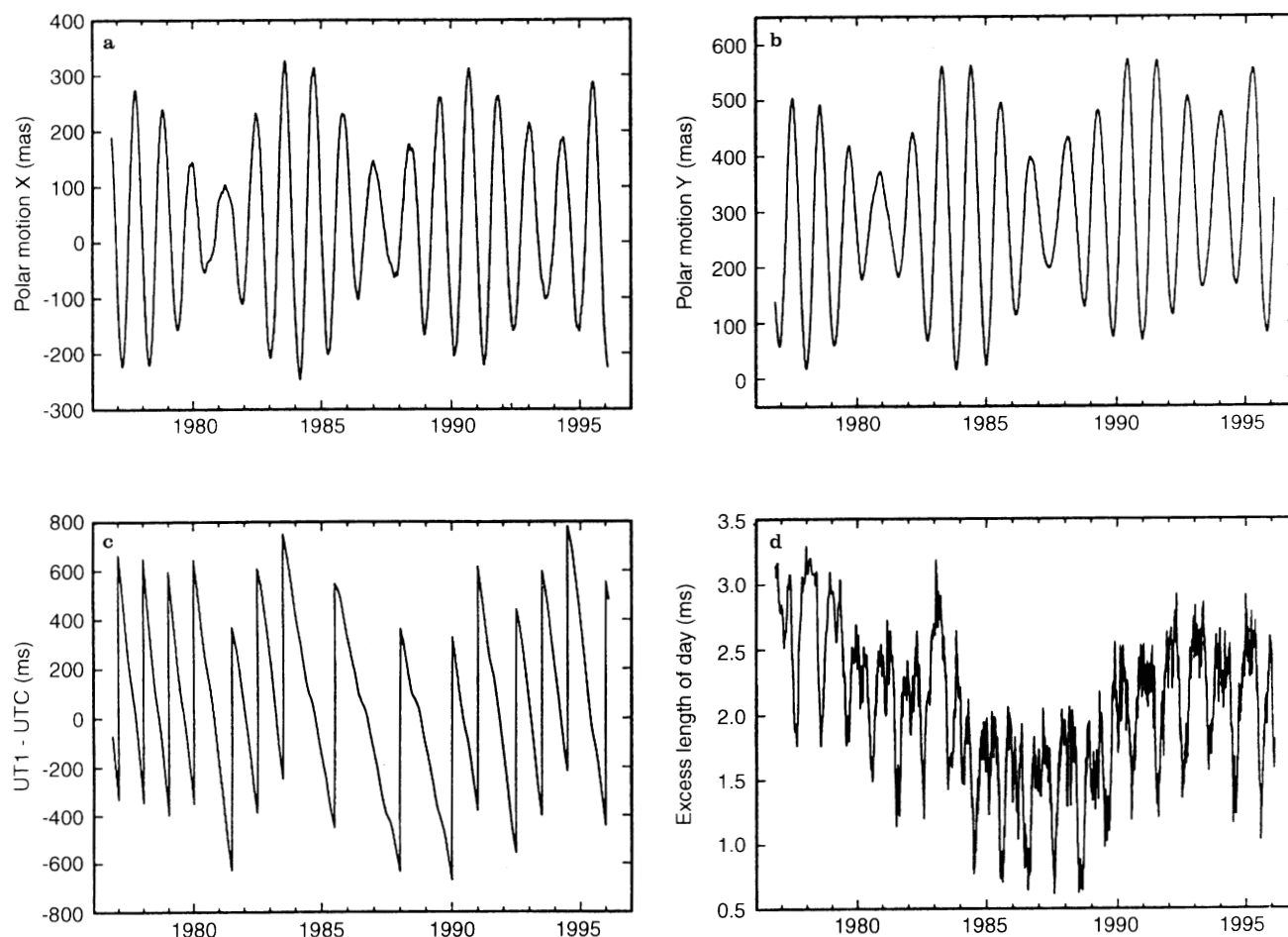
The accuracy of the SPACE95 combined Earth-orientation series is primarily a function of the accuracy of the individual series being combined, although other factors play a role such as the fidelity of the stochastic models employed in the Kalman filter. Not only will the accuracy of the SPACE95 combined series change with time as the accuracy of each individual series changes

**Table 7.** Total adjustments made to series prior to being combined into SPACE95

data set name	bias (mas)			rate (mas/year)			uncertainty scale factor		
LLR (JPL95M01)	VOL		UT0	VOL		UT0	VOL		UT0
McDonald Cluster	−0.206		0.134	−0.568		−0.156	1.630		1.154
CERGA	0.624		−0.055	0.136		−0.012	1.788		1.423
Haleakala	−0.010		−1.454	−0.283		−0.170	1.549		1.692
DSN (JPL95R01)	T		V	T		V	T		V
CA–Spain Cluster	0.178		0.053	0.083		0.059	1.354		1.094
CA–Australia Cluster	0.323		−0.046	−0.072		−0.050	1.371		1.098
SGP/CDP (GLB973f)	T		V	T		V	T		V
Westford–Ft. Davis	11.357		0.576	1.055		−0.090	0.904		0.870
Westford–Mojave	0.283		0.791	−0.028		0.008	2.326		0.954
SGP/CDP (973f)	PMX	PMY	UT1	PMX	PMY	UT1	PMX	PMY	UT1
Multibaseline	−1.203	−2.040	0.480	−0.154	−0.072	−0.121	2.226	1.963	2.192
USNO (N9604)	PMX	PMY	UT1	PMX	PMY	UT1	PMX	PMY	UT1
Multibaseline	−0.153	1.190	−0.705	0.193	0.099	0.043	2.012	1.602	1.783
NOAA (95R02)	PMX	PMY	UT1	PMX	PMY	UT1	PMX	PMY	UT1
IRIS Intensive	–	–	0.748	–	–	−0.072	–	–	0.933
USNO (N9604.INT)	PMX	PMY	UT1	PMX	PMY	UT1	PMX	PMY	UT1
IRIS Intensive	–	–	−0.898	–	–	0.156	–	–	1.840
UTCSR (95L01)	PMX	PMY	UT1	PMX	PMY	UT1	PMX	PMY	UT1
Lageos	−0.010	0.023	–	0.106	−0.049	–	0.849	0.743	–
GPS (SIO93P01)	PMX	PMY	UT1	PMX	PMY	UT1	PMX	PMY	UT1
Scripps	−1.091	−1.516	–	−0.027	−0.018	–	1.956	1.903	–
GPS (JPL95P02)	PMX	PMY	UT1	PMX	PMY	UT1	PMX	PMY	UT1
JPL	−0.194	−0.259	–	0.057	−0.156	–	3.257	2.800	–
GPS (IGS Analysis)	PMX	PMY	UT1	PMX	PMY	UT1	PMX	PMY	UT1
JPL	0.450	0.219	–	−0.027	−0.018	–	12.273	3.542	–

Reference date for bias-rate adjustment is 1993.0

## COMBINED EARTH ORIENTATION SERIES: SPACE95



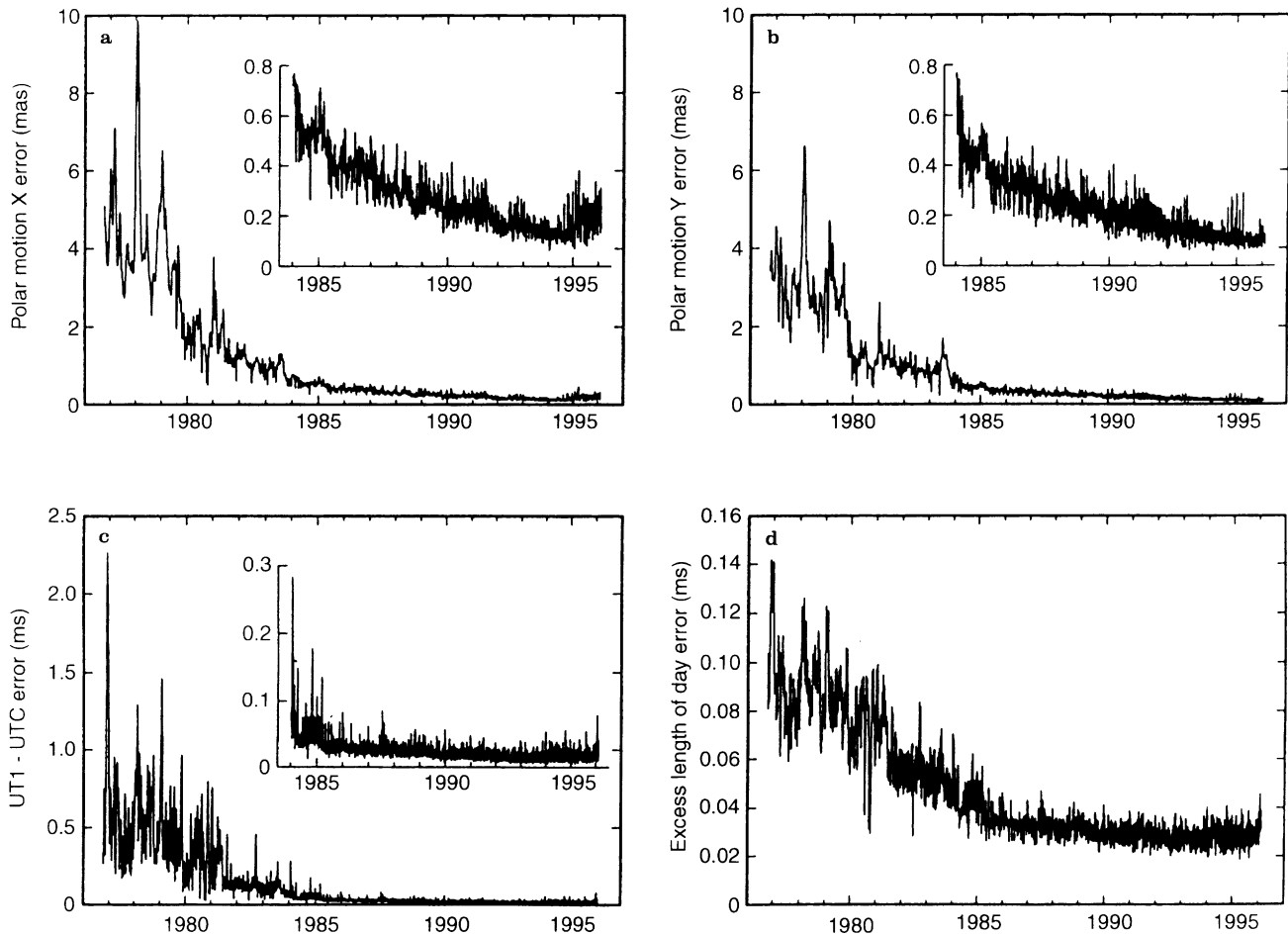
**Fig. 3a-d.** Plots of the values of **a** the  $x$  component of polar motion, **b**  $y$  component of polar motion, **c** UT1-UTC, and **d** excess length of day as given by the Earth-orientation combination SPACE95. The discontinuous changes in the plot of UT1-UTC are caused by the presence of leap seconds. Note that the UT1-UTC series displayed in **c** includes the tidal variations, whereas the LOD series shown in **d** does not

with time, but the accuracy of SPACE95 will also change as the number of series being combined changes. This is particularly evident in the polar motion component of SPACE95, where from Fig. 4a it is seen that the uncertainty of the 1995 values is somewhat larger than that of the 1994 values. From Table 1 it is seen that the SLR series used in generating SPACE95 extends only through 28 January 1995. Thus, fewer series were combined in generating the 1995 polar motion values of SPACE95 than were combined in generating the 1994 values. Also, the particular GPS series (JPL's IGS analysis series) used in generating the 1995 values is different from that [EOP(JPL) 95 P 02] used in generating the 1994 values. The growth in the uncertainty of the SPACE95 polar motion values during 1995 probably results from both these factors: from combining both fewer and different individual series. In particular, from Table 7 it is seen that the uncertainty scale factor (12.273) applied to the  $x$  component of the GPS polar motion series used during 1995 (the IGS Analysis series of JPL) is much larger than that (3.257) applied to the  $x$  component of the GPS series used during 1994 (JPL95P02), reflecting the larger scatter (not shown) in

its residual series. The adjusted uncertainties of the  $x$  component of the GPS polar motion series of 1995 are therefore much larger than those of 1994; consequently, the uncertainties of the  $x$  component of the SPACE95 combined series are greater during 1995 than they are during 1994. This is not as evident in the  $y$  component of SPACE95, since the uncertainty scale factors applied to the  $y$  components of the two GPS polar motion series are more nearly equal (2.800 for the series of 1994 versus 3.542 for that of 1995); hence the adjusted uncertainties of the  $y$  components of the 1994 and 1995 series are more nearly equal, and consequently the SPACE95 uncertainties of 1994 and 1995 are more nearly equal.

This combination, SPACE95, has been submitted to NASA's Crustal Dynamics Data Information System and is available from them by anonymous ftp to the internet address CDDIS.A.GSFC.NASA.GOV (128.183.204.168), where it can be found in the 1995 subdirectory of the JPL subdirectory of the pub directory. SPACE95 has also been submitted to the International Earth Rotation Service (IERS), where it is briefly described and compared to other combined Earth-orientation series in their annual report for 1995 (IERS 1996).

## COMBINED EARTH ORIENTATION SERIES: SPACE95



**Fig. 4a–d.** Plots of the  $1\sigma$  formal errors in the determination of **a** the  $x$  component of polar motion, **b**  $y$  component of polar motion, **c** UT1–UTC, and **d** excess length of day as given by the Earth-orientation combination SPACE95. The insert within panels **a**, **b**, and **c** shows that component's post-1984 uncertainties on an expanded scale with

the same units [milliarcseconds (mas) for PMX and PMY, milliseconds (ms) for UT1–UTC]. The slight increase in the formal errors during 1995 are a reflection of the fact that a number of the individual series combined to form SPACE95 do not extend through the end of 1995, but terminate at the end of 1994 (see Table 1)

## 6 Summary

In this report, an approach currently used at JPL to combine Earth-orientation series has been presented. Many of the complications that arise under this approach are due to the presence of degenerate data types and the desire for the process to be as free as possible from the need to make any subjective decisions. In the absence of degenerate data types, a more conventional Kalman filter based upon the state and measurement covariance matrices, rather than the information matrices, could have been developed and used. However, the degenerate data types provide important information about the Earth's orientation, especially before about 1984, and should therefore be included in any combined Earth-orientation series.

The process of determining bias-rate corrections and uncertainty scale factors is made more complicated by not using a reference series. However, in the absence of any knowledge of the true accuracies of the series being combined, and with the desire to make the combination

process be as free as possible of the need to make any subjective decisions, an approach such as that presented here based upon an intercomparison of the available data sets seems reasonable. But since the uncertainty in the values of the Earth-orientation parameters grows rapidly in the absence of measurements, intercomparing data sets should be done by a technique that can objectively account for this uncertainty growth. The Kalman filter is such a technique, since it contains a model for the process, and in the absence of measurements uses this model to propagate forward in time the state vector and its covariance matrix. The use of complementary smoothings minimizes interpolation error. Furthermore, when using a Kalman filter to combine independent data sets, the time-varying nature of the precision and temporal resolution of the measurements is automatically taken into account, thereby circumventing the need for any subjective decision to be made as to how or when to change the degree of smoothing applied as the properties of the measured series change.

**Acknowledgements.** We thank K. Deutsch, D. Hernquist, D. Morabito, and S. Mulligan for their help in developing the Kalman Earth-orientation filter. We also thank all those involved in taking and reducing the space-geodetic Earth-orientation measurements that have been combined into SPACE95. This study would not have been possible without their considerable efforts. The work described in this paper was performed at the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration.

## References

- Archinal BA (1988) Combination of data from different space geodetic systems for the determination of Earth rotation parameters. In: Babcock AK, Wilkins GA (eds) *The Earth's rotation and reference frames for geodesy and geodynamics*. Reidel, Dordrecht Holland, pp 233–239
- Babcock AK (1988) Recent improvements in the U.S.N.O. combined solution for Earth rotation parameters. In: Babcock AK, Wilkins GA (eds) *The Earth's rotation and reference frames for geodesy and geodynamics*. Reidel, Dordrecht Holland, pp 241–245
- Barnes RTH, Hide R, White AA, Wilson CA (1983) Atmospheric angular momentum fluctuations, length-of-day changes and polar motion. *Proc R Soc London A* 387: 31–73
- Beutler G, Hein GW, Melbourne WG, Seeber G (eds) (1996) *GPS trends in precise terrestrial, airborne, and spaceborne applications*. Springer, Berlin Heidelberg New York
- Bierman GJ (1977) *Factorization methods for discrete sequential estimation*. Academic Press, New York
- Blewitt G (1993) Advances in global positioning system technology for geodynamics investigations: 1978–1992. In: Smith DE, Turcotte DL (eds) *Contributions of space geodesy to geodynamics: technology*. *Geodyn Ser* 25, AGU, Washington DC, pp 195–213
- Bock Y, Leppard N (eds) (1990) *Global positioning system: an overview*. Springer, Berlin Heidelberg New York
- Bock Y, Fang P, Stark K (1993) 1991–1993 SIO polar motion series. In: Charlot P (ed) *Earth orientation, reference frames, and atmospheric excitation functions submitted for the 1992 IERS Annual Report*. IERS Tech Note 14, Obs de Paris, Paris, pp P43–P44
- Brzezinski A (1990) On polar motion equations applied for analysis of the short-term atmospheric excitation. In: Boucher C, Wilkins GA (eds) *Earth rotation and coordinate reference frames*. Springer, Berlin Heidelberg New York, pp 82–89
- Brzezinski A (1992) Polar motion excitation by variations of the effective angular momentum function: considerations concerning deconvolution problem. *Manuscr Geod* 17: 3–20
- Brzezinski A (1994) Polar motion excitation by variations of the effective angular momentum function, II: extended model. *Manuscr Geod* 19: 157–171
- Carter WE (1979) Project POLARIS: a status report. In: *Radio interferometry techniques for geodesy*. NASA Conference Publication 2115, pp 455–460
- Carter WE, Robertson DS (1984) IRIS Earth rotation and polar motion results. Paper presented at Int Symp Space Techniques for Geodynamics, Geod and Geophys, Res Inst Hung Acad Sci and the IAG/COSPAR Joint Comm. on the Int Coord of Space Techniques for Geod and Geodyn, Sopron Hungary
- Carter WE, Robertson DS (1986a) Projects POLARIS and IRIS: monitoring polar motion and UT1 by very long baseline interferometry. In: Anderson AJ, Cazenave A (eds) *Space geodesy and geodynamics*. Academic Press, Orlando Florida, pp 269–279
- Carter WE, Robertson DS (1986b) Accurate Earth orientation time-series from VLBI observations. In: Cazenave A (ed) *Earth rotation: solved and unsolved problems*. Reidel, Dordrecht Holland, pp 61–67
- Carter WE, Strange WE (1979) The national geodetic survey project: “POLARIS”. *Tectonophysics* 52: 39–46
- Carter WE, Robertson DS, Abell MD (1979) An improved polar motion and Earth rotation monitoring service using radio interferometry. In: McCarthy DD, Pilkington JD (eds) *Time and the Earth's rotation*. Reidel, Dordrecht Holland, pp 191–197
- Carter WE, Robertson DS, Pettey JE, Tapley BD, Schutz BE, Eanes RJ, Lufeng M (1984) Variations in the rotation of the Earth. *Science* 224: 957–961
- Carter WE, Robertson DS, MacKay JR (1985) Geodetic radio interferometric surveying: applications and results. *J Geophys Res* 90: 4577–4587
- Carter WE, Robertson DS, Nothnagel A, Nicolson GD, Schuh H, Campbell J (1988) IRIS-S: extending geodetic very long baseline interferometry observations to the Southern Hemisphere. *J Geophys Res* 93: 14947–14953
- Christodoulidis DC, Smith DE, Kolenkiewicz R, Klosko SM, Torrence MH, Dunn PJ (1985) Observing tectonic plate motions and deformations from satellite laser ranging. *J Geophys Res* 90: 9249–9263
- Clark TA, Corey BE, Davis JL, Elgered G, Herring TA, Hinteregger HF, Knight CA, Levine JJ, Lundqvist G, Ma C, Nesman EF, Phillips RB, Rogers AEE, Ronnang BO, Ryan JW, Schupler BR, Shaffer DB, Shapiro II, Vandenberg NR, Webber JC, Whitney AR (1985) Precision geodesy using the Mark-III very-long-baseline interferometer system. *IEEE Trans Geosci Rem Sens* GE-23: 438–449
- Clark TA, Gordon D, Himwich WE, Ma C, Mallama A, Ryan JW (1987) Determination of relative site motions in the western United States using Mark III very long baseline interferometry. *J Geophys Res* 92: 12741–12750
- Cohen SC, Smith DE (1985) LAGEOS scientific results: introduction. *J Geophys Res* 90: 9217–9220
- Dickey JO, Newhall XX, Williams JG (1985) Earth orientation from lunar laser ranging and an error analysis of polar motion services. *J Geophys Res* 90: 9353–9362
- Dickey JO, Bender PL, Faller JE, Newhall XX, Ricklefs RK, Ries JG, Shelus PJ, Veillet C, Whipple AL, Wiant JR, Williams JG, Yoder CF (1994) Lunar laser ranging: a continuing legacy of the Apollo program. *Science* 265: 482–490
- Dickman SR (1993) Dynamic ocean-tide effects on Earth's rotation. *Geophys J Int* 112: 448–470
- Eanes RJ, Watkins MM (1995) Earth orientation and site coordinates from the Center for Space Research solution. In: Charlot P (ed) *Earth orientation, reference frames, and atmospheric excitation functions submitted for the 1994 IERS annual report*. IERS Tech Note 19, Obs de Paris, Paris, pp L7–L11
- Eubanks TM, Steppe JA (1988) The long-term stability of VLBI Earth orientation measurements. In: Reid MJ, Moran JM (eds) *The impact of VLBI on astrophysics and geophysics*. Reidel, Dordrecht Holland, pp 369–370
- Eubanks TM, Roth MG, Esposito PB, Steppe JA, Callahan PS (1982) An analysis of JPL TEMPO Earth orientation results. In: *Proc IAG Symp No. 5: Geodetic Applications of Radio Interferometry*, NOAA Technical Report NOS 95 NGS 24, Nat Geod Survey, Rockville MD, pp 81–90
- Eubanks TM, Steppe JA, Spieth MA (1985a) The accuracy of radio interferometric measurements of Earth rotation. In: Posner E (ed) *The telecommunications and data acquisition progress report 42–80: October–December 1984*, Jet Propulsion Lab, Pasadena Calif, pp 229–235
- Eubanks TM, Steppe JA, Dickey JO, Callahan PS (1985b) A spectral analysis of the Earth's angular momentum budget. *J Geophys Res* 90: 5385–5404
- Eubanks TM, Archinal BA, Carter MS, Josties FJ, Matsakis DN, McCarthy DD (1994) Earth orientation results from the U.S. Naval Observatory VLBI program. In: Charlot P (ed) *Earth orientation, reference frames and atmospheric excitation functions submitted for the 1993 IERS annual report*. IERS Tech Note 17, Obs de Paris, Paris, pp R65–R79

- Fanselow JL, Thomas JB, Cohen EJ, MacDoran PF, Melbourne WG, Mulhall BD, Purcell GH, Rogstad DH, Skjerve LJ, Spitzmesser DJ (1979) Determination of UT1 and polar motion by the Deep Space Network using very long baseline interferometry. In: McCarthy DD, Pilkington JD (eds) Time and the Earth's rotation. Reidel, Dordrecht Holland, pp 199–209
- Freedman AP, Steppe JA, Dickey JO, Eubanks TM, Sung L-Y (1994) The short-term prediction of universal time and length of day using atmospheric angular momentum. *J Geophys Res* 99: 6981–6996
- Gelb A (ed) (1974) Applied optimal estimation. MIT Press, Cambridge Mass
- Gross RS (1995) A combination of EOP measurements: SPACE94. In: Charlot P (ed) Earth orientation, reference frames, and atmospheric excitation functions submitted for the 1994 IERS annual report. IERS Tech Note 19, Obs de Paris, Paris, pp C1–C4
- Gross RS (1996a) Combinations of Earth orientation measurements: SPACE94, COMB94, and POLE94. *J Geophys Res* 101: 8729–8740
- Gross RS (1996b) A Combination of EOP measurements: SPACE95. Summarized in: 1995 IERS Annual Report, Obs de Paris, Paris, pp III1
- Guinot B (1970) Short-period terms in universal time. *Astron Astrophys* 8: 26–28
- Heflin M, Watkins M, Jefferson D, Webb F, Zumberge J (1995) Coordinates, velocities, and EOP from the Jet Propulsion Laboratory using GPS. In: Charlot P (ed) Earth orientation, reference frames, and atmospheric excitation functions submitted for the 1994 IERS annual report, IERS Tech Note 19, Obs de Paris, Paris, pp P25–P28
- Hernquist DC, Eubanks TM, Steppe JA (1984) A Kalman filter and smoother for Earth rotation (abstract). Spring Meeting Suppl., EOS Trans AGU 65(16): 187
- Herring TA (1993) Diurnal and semidiurnal variations in Earth rotation. In: Singh RP, Feissel M, Tapley BD, Shum CK (eds) Observations of Earth from space. Adv Space Res 13. Pergamon, Oxford, pp (11)281–(11)290
- Herring TA, Dong D (1994) Measurement of diurnal and semidiurnal rotational variations and tidal parameters of Earth. *J Geophys Res* 99: 18051–18071
- Hofmann-Wellenhof B (1993) Global positioning system: theory and practice (2nd edn). Springer, Berlin Heidelberg New York
- IERS (1995) 1994 IERS annual report. Obs de Paris, Paris
- IERS (1996) 1995 IERS annual report. Obs de Paris, Paris
- Lambeck K (1988) Geophysical geodesy: the slow deformations of the Earth. Oxford University Press, Oxford
- Langley RB, King RW, Shapiro II (1981) Earth rotation from lunar laser ranging. *J Geophys Res* 86: 11913–11918
- Ma C, Ryan JW, Gordon D, Caprette DS, Himwich WE (1993) Reference frames from CDP VLBI data. In: Smith DE, Turcotte DL (eds) Contributions of space geodesy to geodynamics: earth dynamics, geodyn. ser., vol. 24. AGU, Washington DC, pp 121–145
- Ma C, Ryan JW, Caprette DS (1994) NASA space geodesy program – GSFC data analysis – 1993: VLBI geodetic results 1979–92. NASA tech memo 104605, NASA, Washington DC
- Ma C, Ryan JW, Gordon D (1995) Site positions and velocities, source positions, and Earth orientation parameters from the NASA Space Geodesy Program-GSFC: solution GLB979f. In: Charlot P (ed) Earth orientation, reference frames, and atmospheric excitation functions submitted for the 1994 IERS annual report, IERS Tech Note 19, Obs de Paris, Paris, pp R13–R20
- McCarthy DD, Luzum BJ (1991) Combination of precise observations of the orientation of the Earth *Bull Geod* 65: 22–27
- McCarthy DD, Luzum BJ (1996) Using GPS to determine Earth orientation. In: Beutler G, Hein GW, Melbourne WG, Seeber G (eds) GPS trends in precise terrestrial, airborne, and spaceborne applications. Int Assoc Geod Symp 115. Springer, Berlin Heidelberg New York pp 52–58
- Morabito DD, Eubanks TM, Steppe JA (1988) Kalman filtering of Earth orientation changes. In: Babcock A, Wilkins GA (eds) The Earth's rotation and reference frames for geodesy and geodynamics. Reidel, Dordrecht Holland, pp 257–267
- Moritz H, Muller II (1988) Earth rotation: theory and observation. Ungar, New York
- Mulholland JD (1980) Scientific advances from ten years of lunar laser ranging. *Rev Geophys Space Phys* 18: 549–564
- Nahi NE (1969) Estimation theory and applications. Wiley, New York
- Newhall XX, Williams JG, Dickey JO (1988) Earth rotation from lunar laser ranging. In: Babcock A, Wilkins GA (eds) The Earth's rotation and reference frames for geodesy and geodynamics. Reidel, Dordrecht Holland, pp 159–164
- Newhall XX, Williams JG, Dickey JO (1995) Earth rotation (UT0–UTC and variation of latitude) from lunar laser ranging. In: Charlot P (ed) Earth orientation, reference frames, and atmospheric excitation functions submitted for the 1994 IERS annual report. IERS Tech Note 19, Obs de Paris, Paris, pp M7–M12
- Preisig JR (1992) Polar motion, atmospheric angular momentum excitation and earthquakes – correlations and significance *Geophys J Int* 108: 161–178
- Ray JR, Abell MD, Carter WE, Dillinger WH, Morrison ML (1995) NOAA Earth orientation and reference frame results derived from VLBI observations: final report. In: Charlot P (ed) Earth orientation, reference frames, and atmospheric excitation functions submitted for the 1994 IERS annual report. IERS Tech Note 19, Obs de Paris, Paris, pp R33–R38
- Robertson DS, Carter WE (1982) Earth rotation information derived from MERIT and POLARIS VLBI observations. In: Calame O (ed) High-precision earth rotation and earth-moon dynamics. Reidel, Hingham Mass, pp 97–122
- Robertson DS, Carter WE (1985) Earth orientation determinations from VLBI observations. In: Proc Int Conf Earth rotation and the terrestrial reference frame, Vol. 1. Ohio State University, Columbus, pp 296–306
- Robertson DS, Carter WE, Campbell J, Schuh H (1985) Daily Earth rotation determinations from IRIS very long baseline interferometry. *Nature* 316: 424–427
- Robertson DS, Carter WE, Fallon FW (1988) Earth rotation from the IRIS project. In: Reid MJ, Moran JM (eds) The impact of VLBI on astrophysics and geophysics. Reidel, Dordrecht Holland, pp 391–400
- Ryan JW, Clark TA, Coates RJ, Ma C, Wildes WT, Gwinn CR, Herring TA, Shapiro II, Corey BE, Counselman CC, Hinteregger HF, Rogers AEE, Whitney AR, Knight CA, Vandenberg NR, Pigg JC, Schupler BR, Rönnäng BO (1986) Geodesy by radio interferometry: determinations of baseline vector, Earth rotation, and solid Earth tide parameters with the Mark I very long baseline radio interferometry system. *J Geophys Res* 91: 1935–1946
- Ryan JW, Ma C, Caprette DS (1993) NASA space geodesy program – GSFC data analysis – 1992: final report of the crustal dynamics project; VLBI geodetic results 1979–91. NASA Tech Mem 104572, NASA, Washington, DC
- Shapiro II (1983) Use of space techniques for geodesy. In: Kana-mori H, Boschi E (eds) earthquakes: observation, theory and interpretation. North Holland, Amsterdam, pp 530–568
- Smith DE, Christodoulidis DC, Kolenkiewicz R, Dunn PJ, Klosko SM, Torrence MH, Fricke S, Blackwell S (1985) A global geodetic reference frame from LAGEOS ranging (SL5.1AP). *J Geophys Res* 90: 9221–9233
- Smith DE, Kolenkiewicz R, Dunn PJ, Robbins JW, Torrence MH, Klosko SM, Williamson RG, Pavlis EC, Douglas NB, Fricke SK (1990) Tectonic motion and deformation from satellite laser ranging to LAGEOS. *J Geophys Res* 95: 22013–22041
- Smith DE, Kolenkiewicz R, Dunn PJ, Klosko SM, Robbins JW, Torrence MH, Williamson RG, Pavlis EC, Douglas NB, Fricke SK (1991) LAGEOS geodetic analysis – SL7.1. NASA Tech Mem 104549, Goddard Space Flight Center, Greenbelt Md

- Smith DE, Kolenkiewicz R, Nerem RS, Dunn PJ, Torrence MH, Robbins JW, Klosko SM, Williamson RG, Pavlis EC (1994) Contemporary global horizontal crustal motion. *Geophys J Int* 119: 511–520
- Sovers OJ, Thomas JB, Fanselow JL, Cohen EJ, Purcell Jr GH, Rogstad DH, Skjerve LJ, Spitzmesser DJ (1984) Radio interferometric determination of intercontinental baselines and Earth orientation utilizing Deep Space Network antennas: 1971 to 1980. *J Geophys Res* 89: 7597–7607
- Steppe JA, Oliveau SH, Sovers OJ (1995) Earth rotation parameters from DSN VLBI: 1995. In: Charlot P (ed) Earth orientation, reference frames, and atmospheric excitation functions submitted for the 1994 IERS annual report, IERS Tech Note 19. Obs de Paris, Paris, pp R21–R27
- Stolz A, Bender PL, Faller JE, Silverberg EC, Mulholland JD, Shellus PJ, Williams JG, Carter WE, Currie DG, Kaula WM (1976) Earth rotation measured by lunar laser ranging. *Science* 193: 997–999
- Tapley BD, Schutz BE, Eanes RJ (1985) Station coordinates, baselines, and Earth rotation from LAGEOS laser ranging: 1976–1984. *J Geophys Res* 90: 9235–9248
- Tapley BD, Schutz BE, Eanes RJ, Ries JC, Watkins MM (1993) Lageos laser ranging contributions to geodynamics, geodesy, and orbital dynamics. In: Smith DE, Turcotte DL (eds) Contributions of space geodesy to geodynamics: earth dynamics. *Geodyn Ser*, 24, AGU, Washington DC, pp 147–173
- Vicente RO, Wilson CR (1986) Polar motion estimates from linear combinations of independent series. *Astron Nachr* 307: 137–147
- Whipple AL (1993) Dynamics of the Earth-Moon system. In: Singh RP, Feissel M, Tapley BD, Shum CK (eds) Observations of Earth from Space. *Adv Space Res* 13, Pergamon, Oxford, pp (11)213–(11)219
- Williams JG, Newhall XX, Dickey JO (1993) Lunar laser ranging: geophysical results and reference frames. In: Smith DE, Turcotte DL (eds) Contributions of space geodesy to geodynamics: earth dynamics. *Geodyn Ser* 24, AGU, Washington DC, pp 83–88
- Wilson CR, Vicente RO (1988) The combination of polar motion observations. In: Babcock AK, Wilkins GA (eds) The Earth's rotation and reference frames for geodesy and geodynamics. Reidel, Dordrecht Holland, pp 247–255
- Yoder CF, Williams JG, Parke ME (1981) Tidal variations of Earth rotation. *J Geophys Res* 86: 881–891
- Zumberge JF, Heflin MB, Jefferson DC, Watkins MM, Webb FH (1995) Jet propulsion laboratory IGS analysis center 1994 annual report. In: Zumberge JF, Liu R, Neilan RE (eds) Int GPS Serv Geodyn 1994 Annual Report, IGS Central Bureau, JPL, pp 185–196